On Slant Submanifolds of a Conformal Kenmotsu Manifold

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Abstract: The object of the present paper is to study slant submanifolds of conformal Kenmotsu manifold. In particular, we characterize three-dimensional slant submanifolds of a conformal Kenmotsu manifold via covariant derivative of P^2, P and F, where P is the tangent projection and F is the normal projection over the submanifold of a conformal Kenmotsu manifold.

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1. **Introduction**

Libermann [14], introduced the notion of locally conformal Kähler manifold as follows: Let (M, J, g) be a Hermitian manifold of even dimension $2n$, where J denotes the almost complex structure and g the Hermitian metric. Then, (M, J, g) is called a locally conformal Kähler manifold if for each point u of M, there is an open neighborhood U of u and a positive function $f : M \to R$ on U such that $g = exp(-f)g|U$ is Kählerian. Further, a Hermitian manifold (M, J, g) is globally conformal Kähler manifold if $U = M$. The contact version of conformal changes are given by Vaismann [22] and these are the one of the sixteen classes of almost Hermitian manifolds. Submanifolds of locally conformal Kähler manifold and conformal contact manifold have been studied by several authors (See, [1]-[3], [19, 20]).

On the other hand, Chen [7] introduced the notion of slant submanifold for an almost Hermitian manifold, as a generalization of both holomorphic and totally real submanifolds.

Later, the study of slant submanifolds was enriched by the authors of [6, 9, 10, 12, 15, 12, 16, 18, 19] and many others.

Recently, Abdi and Abedi [1] introduced conformal Kenmotsu manifold and studied invariant, anti-invariant and CR-submanifolds of it [1]-[2].

The paper is organized as follows: In subsections of section 2, we recall the notion Kenmotsu manifold, submanifold and slant submanifold of almost contact manifold. In section 3, we give a brief account of conformal Kenmotsu manifold. Section 4 is devoted to the study of slant submanifolds of conformal Kenmotsu manifold. Section 5 deals with the study of characterization of three dimensional slant submanifolds of conformal Kenmotsu manifold.

2. Preliminaries

2.1. **Kenmotsu manifold.** A $(2n+1)$-dimensional differentiable manifold $\tilde{M}$ is said to have an almost contact structure $(\tilde{\phi}, \tilde{\xi}, \tilde{\eta})$ if

$$\tilde{\phi}^2 = -I + \tilde{\eta} \otimes \tilde{\xi}, \quad \tilde{\eta}(\tilde{\xi}) = 1, \tag{2.1}$$

which imply that $\tilde{\phi}\tilde{\xi} = 0$, and $\tilde{\eta} \cdot \tilde{\phi} = 0$, where $\tilde{\phi}$ is a tensor field of type $(1, 1)$, $\tilde{\xi}$ a vector field and $\tilde{\eta}$ is a 1-form on $\tilde{M}$.
Further, $\tilde{M}$ is said to have an almost contact metric structure $(\tilde{\phi}, \tilde{\xi}, \tilde{\eta}, \tilde{g})$, if $(\tilde{\phi}, \tilde{\xi}, \tilde{\eta})$ together with Riemannian metric $\tilde{g}$ satisfying the condition

$$\tilde{g}(\tilde{\phi}X, \tilde{\phi}Y) = \tilde{g}(X, Y) - \tilde{\eta}(X)\tilde{\eta}(Y) \tag{2.2}$$

for all vector fields X, Y on $\tilde{M}$. The characteristic vector field $\tilde{\xi}$ is unitary with respect to $\tilde{g}$ and orthogonal to the distribution D.
If, moreover

$$(\tilde{\nabla}_X\tilde{\phi})Y = -\tilde{\eta}(Y)\tilde{\phi}X - \tilde{g}(X, \tilde{\phi}Y)\tilde{\xi}, \tag{2.3}$$

$$\tilde{\nabla}_X\tilde{\xi} = X - \tilde{\eta}(X)\tilde{\xi}, \tag{2.4}$$

where $\tilde{\nabla}$ denotes the Riemannian connection of $\tilde{g}$, then $\tilde{M}(\tilde{\phi}, \tilde{\xi}, \tilde{\eta}, \tilde{g})$ is called an almost Kenmotsu manifold [13].

2.2. **Submanifold.** Assume M is a submanifold of an almost contact metric manifold $\tilde{M}$. Let g and ∇ be the induced Riemannian metric and connections of M, respectively. Then the Gauss and Weingarten formulae are given respectively, by

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X,Y), \quad \tilde{\nabla}_X V = -A_V X + \nabla^{\perp}_X V, \tag{2.5}$$

for all X, Y on TM and $V \in T^{\perp}M$, where $\nabla^{\perp}$ is the normal connection and A is the shape operator of M with respect to the unit normal vector V. The second fundamental form σ and the shape operator A are related by:

$$g(\sigma(X,Y), V) = g(A_V X, Y). \tag{2.6}$$

Let R and $\tilde{R}$ denote the curvature tensor of M and $\tilde{M}$, then, the Gauss and Ricci equations are given by

$$\tilde{g}(\tilde{R}(X,Y)Z,W) = g(R(X,Y)Z,W) - g(\sigma(X,W),\sigma(Y,Z)) + g(\sigma(X,Z),\sigma(Y,W)), \tag{2.7}$$
$$\tilde{g}(\tilde{R}(X,Y)N_1,N_2) = g(R^{\perp}(X,Y)N_1,N_2) - g([A_1,A_2]X,Y),$$

for all $X, Y, Z, W \in TM$, $N_1, N_2 \in T^{\perp}M$ and A_1, A_2 are shape operators corresponding to N_1, N_2 respectively.

For each $x \in M$ and $X \in T_xM$, we decompose ϕX into tangential and normal components as:

$$\phi X = PX + FX, \tag{2.8}$$

where, P is an endomorphism and F is normal valued 1-form on T_xM. Similarly, for any $V \in T^{\perp}_x M$, we decompose ϕV into tangential and normal components as:

$$\phi V = pV + fV, \tag{2.9}$$

where, p is a tangent valued 1-form and f is an endomorphism on $T^{\perp}_x M$.

2.3. **Slant submanifolds of an almost contact metric manifold.** Let $\tilde{M}$ be a $2n+1$-dimensional almost contact metric manifold with structure $(\tilde{\phi}, \tilde{\xi}, \tilde{\eta}, \tilde{g})$. We say that an immersed submanifold M of $\tilde{M}$ is slant in $\tilde{M}$ if, for any $x \in M$ and $X \in T_xM$ such that X, ξ are linearly independent, the angle $\theta(x) \in [0, \frac{\pi}{2}]$ between ϕX and T_xM is a constant θ, that is θ does not depend on the choice of X and $x \in M$. θ is called the slant angle of M in $\tilde{M}$. Invariant and anti-invariant submanifolds are slant submanifolds with slant angle θ equal to

0 and $\frac{\pi}{2}$, respectively [8]. A slant submanifold which is neither invariant nor anti-invariant is called a proper slant submanifold.

We mention the following results for later use.

Theorem 2.1. [6] *Let M be a submanifold of an almost contact metric manifold $\tilde{M}$ such that $\xi \in TM$. Then, M is slant if and only if there exists a constant $\lambda \in [0,1]$ such that*

$$P^2 = -\lambda(I - \eta \otimes \xi). \tag{2.10}$$

Further more, if θ is the slant angle of M, then $\lambda = cos^2\theta$.

Corollary 2.1. [6] *Let M be a slant submanifold of an almost contact metric manifold $\tilde{M}$ with slant angle θ. Then, for any $X, Y \in TM$, we have*

$$g(PX, PY) = cos^2\theta(g(X, Y) - \eta(X)\eta(Y)), \tag{2.11}$$

$$g(FX, FY) = sin^2\theta(g(X, Y) - \eta(X)\eta(Y)). \tag{2.12}$$

Lemma 2.1. [15] *Let M be a slant submanifold of an almost contact metric manifold $\tilde{M}$ with slant angle θ. Then, at each point x of M, $Q|D$ has only one eigenvalue $\lambda_1 = -cos^2\theta$.*

Lemma 2.2. [16] *Let M be a 3-dimensional slant submanifold of an almost contact metric manifold $\tilde{M}$. Suppose that M is not anti invariant. If $p \in M$, then in a neighborhood of p, there exist vector fields e_1, e_2 tangent to M, such that ξ, e_1, e_2 is a local orthonormal frame satisfying*

$$Pe_1 = (cos\theta)e_2, \quad Pe_2 = -(cos\theta)e_1. \tag{2.13}$$

3. Conformal Kenmotsu manifold

A $(2n+1)$-dimensional Riemannian manifold $\tilde{M}$ with almost contact metric structure $(\bar{\phi}, \bar{\xi}, \bar{\eta}, \bar{g})$ is called a conformal Kenmotsu manifold [1] if for a smooth function $f : \tilde{M}^{2n+1} \to \mathbb{R}$ so that

$$\tilde{g} = exp(f)\bar{g}, \;\; \tilde{\phi} = \bar{\phi}, \quad \tilde{\eta} = (exp(f))^{\frac{1}{2}}\bar{\eta}, \;\; \tilde{\xi} = (exp(-f))^{\frac{1}{2}}\bar{\xi}.$$

such that $\tilde{M}(\tilde{\phi}, \tilde{\xi}, \tilde{\eta}, \tilde{g})$ is a Kenmotsu manifold.

Let $\tilde{M}$ is a conformal Kenmotsu manifold. Suppose $\tilde{\nabla}$ and $\bar{\nabla}$ are the Riemannian connections on $\tilde{M}$ with respect to $\tilde{g}$ and $\bar{g}$ respectively. Using the Koszul formula, we obtain the following relation between the connections $\tilde{\nabla}$ and $\bar{\nabla}$:

$$\tilde{\nabla}_X Y = \bar{\nabla}_X Y + \frac{1}{2}\{\omega(X)Y + \omega(Y)X - \bar{g}(X,Y)\omega^{\sharp}\}, \tag{3.1}$$

for all $X, Y \in T\tilde{M}$, where $\omega(X) = X(f)$ and $\omega^{\sharp} = gradf$ is a vector field metrically equivalent to 1-form ω, that is $\bar{g}(\omega^{\sharp}, X) = \omega(X)$.

Using (3.1), we obtain the relation between the curvature tensors of $\tilde{M}(\tilde{\phi}, \tilde{\xi}, \tilde{\eta}, \tilde{g})$ and $\tilde{M}(\bar{\phi}, \bar{\xi}, \bar{\eta}, \bar{g})$ as follow:

$$\begin{aligned} exp(-f)(\tilde{R}(X,Y,Z,W)) &= \bar{R}(X,Y,Z,W) + \frac{1}{2}\{B(X,Z)\bar{g}(Y,W) - B(Y,Z) \\ &\quad \bar{g}(X,W) + B(Y,W)\bar{g}(X,Z) - B(X,W)\bar{g}(Y,Z)\} \\ &\quad + \frac{1}{4}\|\omega^{\sharp}\|^2\{\bar{g}(X,Z)\bar{g}(Y,W) - \bar{g}(Y,Z)\bar{g}(X,W)\}, \end{aligned} \tag{3.2}$$

for all $X, Y, Z, W \in T\tilde{M}$, such that $B = \bar{\nabla}\omega - \frac{1}{2}\omega \otimes \omega$. Furthermore, by the relations, (2.7), (2.8) and (3.1) we get

$$\begin{aligned} (\bar{\nabla}_X \bar{\phi})Y &= (exp(f))^{\frac{1}{2}}\{-\bar{g}(X, \bar{\phi}Y)\bar{\xi} - \bar{\eta}(Y)\bar{\phi}X\} \\ &\quad -\frac{1}{2}\{\omega(\bar{\phi}Y)X - \omega(Y)\bar{\phi}X + \bar{g}(X,Y)\bar{\phi}\omega^{\sharp} - \bar{g}(X, \bar{\phi}Y)\omega^{\sharp}\}, \end{aligned} \tag{3.3}$$

$$\bar{\nabla}_X \bar{\xi} = (exp(f))^{\frac{1}{2}}\{X - \bar{\eta}(X)\bar{\xi}\} - \frac{1}{2}\{\omega(\bar{\xi})X - \bar{\eta}(X)\omega^{\sharp}\}, \tag{3.4}$$

for any vector fields X, Y on $\bar{M}$ [1].

Now assume M is a submanifold of a conformal Kenmotsu manifold $\bar{M}$ and ∇, R are the connection, curvature tensor on M, respectively, and g is an induced metric on M. Then, from (3.3), it follows that

$$\begin{aligned} (\nabla_X P)Y &= A_{FY}X + p\sigma(X,Y) + (exp(f))^{\frac{1}{2}}\{-g(X,PY)\xi - \eta(Y)PX\} \\ &\quad -\frac{1}{2}\{\omega(PY)X - \omega(Y)PX + g(X,Y)(P\omega^{\sharp}) - g(X,PY)(\omega^{\sharp})^{\top}\}, \end{aligned} \tag{3.5}$$

$$(\nabla_X F)Y = f\sigma(X,Y) - \sigma(X,PY) + \frac{1}{2}\{\omega(Y)FX - g(X,Y)F\omega^{\sharp} + g(X,PY)\omega^{\sharp\perp}\}, \tag{3.6}$$

where, $g = \bar{g}|M$, $\eta = \bar{\eta}|M$, $\xi = \bar{\xi}|M$ and $\phi = \bar{\phi}|M$.

4. Slant submanifolds of conformal Kenmotsu manifolds

In this section, we characterize slant submanifolds of a conformal Kenmotsu manifold.

Theorem 4.2. *Let M be a slant submanifold of conformal Kenmotsu manifold $\bar{M}$ such that $\omega^\sharp \in T^\perp M$ and $\xi \in TM$. Then Q is parallel if and only if one of the following is true:*
(i) M is anti-invariant;
(ii) $dim(M) \geq 3$;
(iii) M is trivial.

Proof. Let θ be the slant angle of M in $\bar{M}$, then for any $X, Y \in TM$ and by equation (2.8), we infer

$$P^2Y = QY = cos^2\theta(-Y + \eta(Y)\xi). \tag{4.1}$$

Replacing Y by ∇Y in (4.1) imply

$$Q(\nabla_X Y) = cos^2\theta(-\nabla_X Y + \eta(\nabla_X Y)\xi). \tag{4.2}$$

On differentiating (4.1) covariantly with respect to X, yields

$$\nabla_X QY = cos^2\theta(-\nabla_X Y + \eta(\nabla_X Y)\xi + g(Y, \nabla_X \xi)\xi + \eta(Y)\nabla_X \xi). \tag{4.3}$$

From (4.2) from (4.3), we have

$$(\nabla_X Q)Y = cos^2\theta[g(Y, \nabla_X \xi)\xi + \eta(Y)\nabla_X \xi]. \tag{4.4}$$

From (4.4) we can conclude that, if Q is parallel then either $cos(\theta) = 0$ i.e. M is anti-invariant or

$$g(\nabla_X \xi, Y)\xi + \eta(Y)\nabla_X \xi = 0. \tag{4.5}$$

We know $g(\nabla_X \xi, \xi) = 0$, since $g(\nabla_X \xi, \xi) = -g(\xi, \nabla_X \xi)$, which implies $\nabla_X \xi \in D$.
Suppose $\nabla_X \xi \neq 0$, then (4.5) yields $\nabla_X \xi \perp \xi$ and $\nabla_X \xi \perp Y$. Implies $\{\nabla_X \xi, Y, \xi\}$ spans M. Hence $dim(M) \geq 3$.
Suppose $\nabla_X \xi = 0$ i.e. $X = \eta(X)\xi$. Since $TM = D \oplus < \xi >$ and $X \in TM$. Hence M is trivial. □

Now, we state the main result of this section.

Theorem 4.3. *Let M be a slant submanifold of conformal Kenmotsu manifold $\bar{M}$ such that $\xi \in TM$. Then M is slant if and only if*

(1) *The endomorphism $Q|D$ has only one eigen value at each point of M.*

(2) *There exists a function $\lambda : M \to [0,1]$ such that*

$$\begin{aligned}(\nabla_X Q)Y &= \lambda\{(exp(f))^{\frac{1}{2}}[g(X,Y)\xi - \eta(Y)X] \\ &\quad -\frac{1}{2}\{\omega(\xi)g(X,Y)\xi - \eta(X)\omega(Y)\xi + \omega(\xi)\eta(Y)X - \eta(X)\eta(Y)\omega^{\sharp^T}\}\}, \quad (4.6)\end{aligned}$$

for any $X, Y \in TM$. Moreover, if θ is the slant angle of M, then $\lambda = cos^2\theta$.

Proof. Statement 1 follows directly from **Lemma (2.1)**. So, it remains to prove statement 2. Let M be a slant submanifold, then by (4.4) we have

$$(\nabla_X Q)Y = cos^2\theta(g(Y, \nabla_X\xi)\xi + \eta(Y)\nabla_X\xi). \quad (4.7)$$

Putting (3.4) in (4.7), we find (4.6). Conversely, let $\lambda_1(x)$ is the only eigenvalue of $Q|D$ at each point $x \in M$ and $Z \in D$ be a unit eigenvector associated with λ_1, i.e., $QZ = \lambda_1 Z$. Then from statement (2), we have

$$\begin{aligned}X(\lambda_1)Z + \lambda_1\nabla_X Z &= \nabla_X(QZ) \\ &= Q(\nabla_X Z) + \lambda\{(exp(f))^{\frac{1}{2}}[g(X,Z)\xi - \eta(Z)X] - \frac{1}{2}\{\omega(\xi)g(X,Z)\xi \\ &\quad -\eta(X)\omega(Z)\xi + \omega(\xi)\eta(Z)X - \eta(X)\eta(Z)\omega^{\sharp^T}\}\}, \quad (4.8)\end{aligned}$$

for any $X \in TM$. Since both $\nabla_X Z$ and $Q(\nabla_X Z)$ are perpendicular to Z, we conclude that $X(\lambda_1) = 0$. Hence λ_1 is constant. So it remains to prove M is slant. For proving we refer to Theorem (4.3) in [6]. □

5. **Slant submanifolds of dimension three**

This section deals with the characterization of three dimensional slant submanifolds of conformal Kenmotsu manifold.

Theorem 5.4. *Let M be a 3-dimensional proper slant submanifold of a conformal Kenmotsu manifold $\bar{M}$, such that $\xi \in TM$, then*

$$\begin{aligned}(\nabla_X P)Y &= 2(exp(f))^{\frac{1}{2}} g(Y, PX) - \omega(\xi) g(Y, PX) \\ &\quad -\eta(Y)[(exp(f))^{\frac{1}{2}} PX + \frac{1}{2}\{\omega(\xi)PX - \eta(X)\omega^{\sharp^T}\}] \quad (5.1)\end{aligned}$$

for any $X, Y \in TM$ and θ is the slant angle of M.

Proof. Let $X, Y \in TM$ and $u \in M$. Let e_1, e_2, ξ be the orthonormal frame in a neighborhood U of u given by **Lemma (2.2)**. Put $\xi|U = e_3$ and let α_i^j be the structural 1-forms defined by

$$\nabla_X e_i = \sum_{j=1}^{3} \alpha_i^j e_j. \quad (5.2)$$

In view of orthonormal frame e_1, e_2, ξ, we have

$$Y = g(Y, e_1)e_1 + g(Y, e_2)e_2 + \eta(Y)e_3. \quad (5.3)$$

Thus, we get

$$(\nabla_X P)Y = g(Y, e_1)(\nabla_X P)e_1 + g(Y, e_2)(\nabla_X P)e_2 + \eta(Y)(\nabla_X P)e_3. \quad (5.4)$$

Therefore, to obtain $(\nabla_X P)Y$, we have to find $(\nabla_X P)e_1$, $(\nabla_X P)e_2$ and $(\nabla_X P)e_3$. Using (2.13) and (5.2), we obtain

$$\begin{aligned}(\nabla_X P)e_1 &= \nabla_X(Pe_1) - P(\nabla_X e_1) \\ &= \nabla_X((cos\theta)e_2) - P(\alpha_1^1(X)e_1 + \alpha_1^2(X)e_2 + \alpha_1^2(X)e_2) \\ &= (cos\theta)\alpha_2^3(X)e_3. \quad (5.5)\end{aligned}$$

Analogously, we have

$$(\nabla_X P)e_2 = -(cos\theta)\alpha_1^3(X)e_3. \quad (5.6)$$

Moreover, by using (3.4), we obtain

$$\begin{aligned}(\nabla_X P)e_3 &= \nabla_X(Pe_3) - P(\nabla_X e_3) \\ &= -[exp(f)^{\frac{1}{2}} PX + \frac{1}{2}\{\omega(\xi)PX - \eta(X)\omega^{\sharp^T}\}]. \quad (5.7)\end{aligned}$$

Taking account of (5.5)-(5.7) in (5.4), we obtain

$$\begin{aligned}(\nabla_X T)Y &= cos(\theta)\{g(Y,e_1)\alpha_2^3(X)e_3 - g(Y,e_2)\alpha_1^3(X)e_3\}\\ &\quad -\eta(Y)[exp(f)^{\frac{1}{2}}PX + \frac{1}{2}\{\omega(\xi)PX - \eta(X)\omega^{\sharp^T}\}]. \end{aligned} \tag{5.8}$$

Now, we obtain $\alpha_1^3(X)$ and $\alpha_2^3(X)$ as follows:

$$\begin{aligned}\alpha_1^3(X) &= g(\nabla_X e_1, e_3)\\ &= Xg(e_1,e_3) - g(e_1,\nabla_X e_3)\\ &= -(exp(f))^{\frac{1}{2}}[g(e_1,X) - \eta(X)g(e_1,\xi)] + \frac{1}{2}\{\omega(\xi)g(e_1,X) - \eta(X)\omega(e_1)\}, \end{aligned} \tag{5.9}$$

and analogously we have

$$\alpha_2^3(X) = -(exp(f))^{\frac{1}{2}}[g(e_2,X) - \eta(X)g(e_2,\xi)] + \frac{1}{2}\{\omega(\xi)g(e_2,X) - \eta(X)\omega(e_2)\}. \tag{5.10}$$

Substituting (5.9) and (5.10) in (5.8) and after simplification we obtain (5.1). □

From, **Theorems (4.3)** and **(5.4)**, we can state the following:

Corollary 5.2. *Let M be a three dimensional submanifold of a conformal Kenmotsu manifold tangent to ξ. Then the following statements are equivalent:*

(1) *M is slant;*

(2) $(\nabla_X P)Y = 2(exp(f))^{\frac{1}{2}}g(Y,PX) - \omega(\xi)g(Y,PX)$

$$-\eta(Y)[(exp(f))^{\frac{1}{2}}PX + \frac{1}{2}\{\omega(\xi)PX - \eta(X)\omega^{\sharp^T}\}];$$

(3) $(\nabla_X Q)Y = \lambda\{(exp(f))^{\frac{1}{2}}[g(X,Y)\xi - \eta(Y)X]$

$$-\frac{1}{2}\{\omega(\xi)g(X,Y)\xi - \eta(X)\omega(Y)\xi + \omega(\xi)\eta(Y)X - \eta(X)\eta(Y)\omega^{\sharp^T}\}\}.$$

The next result characterizes 3-dimensional slant submanifold in terms of the Weingarten map.

Theorem 5.5. *Let M be a 3-dimensional proper slant submanifold of a conformal Kenmotsu manifold $\bar{M}$, such that $\xi \in TM$. Then, there exists a function $C : M \to [0,1]$ such that*

$$\begin{aligned} A_{FX}Y &= A_{FY}X - 4(exp(f))^{\frac{1}{2}} g(Y,PX) + \eta(Y)[(exp(f))^{\frac{1}{2}} PX \\ &+ \frac{1}{2}\{\omega(\xi)PX - \eta(X)\omega^{\sharp T}\}] - \eta(Y)[(exp(f))^{\frac{1}{2}} PX + \frac{1}{2}\{\omega(\xi)PX - \eta(X)\omega^{\sharp T}\}] \\ &- (exp(f))^{\frac{1}{2}}\{\eta(X)PY - \eta(Y)PX\} + \frac{1}{2}\{\omega(PY)X - \omega(PX)Y - \omega(Y)PX \\ &+ \omega(X)PY - 2g(X,PY)\omega^{\sharp T}\} \end{aligned} \tag{5.11}$$

for any $X, Y \in TM$. Moreover in this case, if θ is the slant angle of M then we have $C = sin^2\theta$.

Proof. Let $X, Y \in TM$ and M is a slant submanifold. From (3.5) and from **Theorem (5.4)**, we have

$$\begin{aligned} p\sigma(X,Y) &= 2(exp(f))^{\frac{1}{2}} g(Y,PX) - \omega(\xi)g(Y,PX) - \eta(Y)[(exp(f))^{\frac{1}{2}} PX + \frac{1}{2}\{\omega(\xi)PX \\ &- \eta(X)\omega^{\sharp T}\}] - A_{FY}X + (exp(f))^{\frac{1}{2}}\{-g(X,PY)\xi - \eta(Y)PX\} \\ &- \frac{1}{2}\{\omega(PY)X - \omega(Y)PX + g(X,Y)P\omega^{\sharp} - g(X,PY)\omega^{\sharp T}\} \end{aligned} \tag{5.12}$$

Now by using the fact symmetry of second fundamental form and metric tensor, we obtain (5.11). □

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