Effect of Variable Gravity Field on Penetrative Convection with Porous medium

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Abstract: ***The effect of vertical through flow and variable gravity field on the onset of penetrative convection simulated via internal heating in a porous medium is studied. Flow in the porous medium is governed by Forchheimer-extended Darcy equation. The boundaries are considered to be rigid, however permeable, and insulated to temperature perturbations. The Eigen value problem is solved using a regular perturbation technique with wave number as a perturbation parameter. The variable gravity parameter, the direction of through flow and the presence of volumetric internal heat source in a porous layer play a decisive role on the stability characteristics of the system. In addition, the influence of Prandtl number arising due to through flow is also emphasized on the stability of the system. It is observed that both stabilizing and destabilizing factors can be enhanced due to the simultaneous presence of a volumetric source, gravity field and vertical through flow so that a more precise control (suppress or augment) of thermal convective instability in a layer of porous medium is possible.***

Keywords: ***Porous medium, Penetrative convection, Gravity field, Regular perturbation method,***

I. INTRODUCTION

A quantitative study is essential to understand the physics of flow behaviour and for obtaining invaluable scale-up information in industrial applications. One of the most important and dominant transport mechanisms in many applications is convection. Recently, there is a rapid change in technology which demands a thorough understanding of the principles of fluid mechanics and knowledge of how to apply them. Bio-medical, aeronautics, civil, mechanical, marine and chemical engineers as well as space researchers, meteorologists, geophysicists and physical oceanographers encounter multitude of complex flow phenomena.

The convective instability in a fluid-saturated porous medium due to a vertical through flow is of relevance in the study of geothermal activities, underground transport of pollutants, gas reservoirs, groundwater pollution and thermal insulation, in the manufacture of composite materials used in aircraft structures and automobile industries, geophysical systems, crystal growth and in flow in biological materials. Copious literature available on this topic is well documented in the book by Nield and Bejan [12]. The in situ processing of energy resources such as coal, oil shale, or geothermal energy often involves the through flow in the porous medium. The importance of buoyancy-driven instability in such systems may become significant when precise processing is required.

II. MATHEMATICAL FORMULATION

The physical configuration consists of an infinite horizontal incompressible fluid-saturated porous layer of thickness d with through flow of vertical velocity W_0 and the *z*-axis pointing vertically upwards opposing the direction of gravity. The both upper and lower boundary is insulated to temperature perturbations. The temperatures of the lower and upper boundaries are taken to be uniform and equal to T_l and T_u respectively, with $T_l > T_u$ .The density of the fluid is given by $\rho = \rho_0\left[1-\alpha\left(T-T_0\right)\right]$ (1)

where α is the positive coefficient of the thermal liquid expansion and ρ_0 is the value at the reference temperature T_0 .

As pointed by Beck, the inclusion of inertial effects in the porous medium by adding $(\vec{V}_m \cdot \nabla)\vec{V}_m$ is not correct for the reasons that this term (i) vanishes identically if the flow is unidirectional and hence cannot represent the inertial effect(increase in drag) in that case, and (ii) raises the order of the differential equation with respect to space derivatives when the base flow is not quiescent which leads to inconsistency with the velocity boundary conditions when the Darcy law is employed. For many naturally occurring porous media, Nield [12] have shown that $|\vec{V}_m|\vec{V}_m$ is the appropriate inertia term in the momentum equation, which implies that the effect of inertia is a drag term quadratic in the velocity $\vec{V}_m$. These aspects are discussed at length by Nield and Bejan [12].Under the circumstances, the Darcy-Forchheimer equation is used to describe the flow in the porous medium to account for inertia effects arising due to vertical throughflow. Thus, the governing equations for the porous layer in the Boussinesq approximation are:

$$\nabla \cdot \vec{V} = 0 \tag{2}$$

$$\frac{\rho_0}{\phi}\left(\frac{\partial \vec{V}}{\partial t} + \frac{C_b}{\sqrt{\kappa}}\vec{V}\left|\vec{V}\right|\right) = -\nabla p - \frac{\mu}{\kappa}\vec{V} + \rho\vec{g} \tag{3}$$

$$A\frac{\partial T}{\partial t} + \left(\vec{V}\cdot\nabla\right)T = k\nabla^2 T + q \tag{4}$$

where, following Chen and Chen [4] it has been assumed that the gravity vector $\vec{g}$ varies linearly with distance. That is,

$$\vec{g} = -g_0\left(1+\lambda z\right)\hat{k} \tag{5}$$

Where λ, the variable gravity coefficient is assumed to be a constant.

In the above equations, $\vec{V} = (u, v, w)$ is the velocity vector, T is the temperature, μ is the dynamic viscosity, p is the pressure, q is the heat source in the porous layer, C_b is the form drag constant, A is the ratio of heat capacities, ρ_0 is the reference fluid density, κ and ϕ permeability and porosity of the porous medium. The basic steady state solution is of the form

$$(u, v, w, p, T) = \left(0, 0, w_0, p_b(z), T_b(z)\right) \tag{6}$$

where the subscript b denotes the basic state and the temperature distributions in the basic state is given by

$$T_b(z) = T_l + \frac{qd}{W_0}\left[\left(\frac{z}{d} + \frac{1-e^{W_0 z/\kappa}}{e^{W_0 d/\kappa}-1}\right)\right] - \left(T_l - T_u\right)\left(\frac{1-e^{W_0 z/\kappa}}{e^{W_0 d/\kappa}-1}\right). \tag{7}$$

In order to investigate the stability of the basic solution, infinitesimal disturbances are introduced in the form

$$(u, v, w, p, T) = \left(0, 0, w_0, p_b(z), T_b(z)\right) + \left(u', v', w', p', T'\right) \tag{8}$$

where the primed quantities are the perturbation and assumed to be small. Now Eq.(8)is substituted in Eqs.(2)−(4) and linearized in the usual manner. The pressure term is eliminated from Eq. (3) by taking curl twice on this equation and only the vertical component is retained. The variables are then nondimensionalized using d, d^2/κ, κ/d and $T_l - T_u$ units of length, time, velocity, and temperature. The non-dimensional disturbance equations are now given by

$$\left[\frac{Da}{\phi \Pr}\left(\frac{\partial}{\partial t}+C_{bm}\left|Pe\right|\right)+1\right]\nabla^2 w = R\left(1+\lambda z\right)\nabla^2 w \qquad (9)$$

$$\left[\frac{\partial}{\partial t}+Pe\frac{\partial}{\partial z}-\nabla^2\right]T=\left[\frac{2Ns}{Pe}+(Pe+2Ns)\frac{e^{Pez}}{\left(1-e^{Pe}\right)}\right]w \qquad (10)$$

where $R=\alpha g_0\left(T_l-T_u\right)d^3/\nu\kappa$ is the Rayleigh number, $Ns=qd^2/2\kappa\left(T_l-T_u\right)$ is the dimensionless heat source strength, $Da=K/d^2$ is the Darcy number and $C_{bm}=C_b d/\sqrt{K}$ is the non dimensional group. The principle of exchange instabilities holds good even for the present configuration as well, hence the time derivatives will be dropped conveniently from Eqs. (9) and (10).Then performing a normal mode expansion and seek solutions for the dependent variables as

$$\left(w,T\right)=\left[W\left(z\right),\Theta\left(z\right)\right]\exp\left[i\left(lx+my\right)\right] \qquad (11)$$

and substituting them in Eqs. (9) and (10) (with $\partial/\partial t$ = 0), we obtain the following ordinary differential equations

$$(1+G)\left(D^2-a^2\right)\,W=-R\,a^2\,(1+\lambda z)\Theta \qquad (12)$$

$$\left(D^2-PeD-a^2\right)\Theta=f(z)W \qquad (13)$$

Where W is the amplitude of perturbed vertical velocity and Θ is the amplitude of perturbed temperature. In the above equations, $D=d/dz$, $a=\sqrt{l^2+m^2}$ is the overall horizontal wave numbers, $G=C_{bm}\,Da\left|Pe\right|/\phi Pr_m$ is the non dimensional groups and

$$f(z)=\frac{2Ns}{Pe}+\frac{\left(Pe+2\,Ns\right)}{\left(1-e^{Pe}\right)}e^{Pez}. \qquad (14)$$

The boundary conditions take the form

$$W=0\ \ or\ \ DW=0\quad at\ \ z=0,1 \qquad (15)$$

$$D\Theta=0\quad at\ \ z=0,\,1. \qquad (16)$$

III. Solution Procedure

When the boundaries are insulated to temperature perturbations, the critical wave number is vanishingly small. Hence, the solution of eigen value problem may be obtained in a closed form using regular perturbation technique with wave number a as the perturbation parameter. The variables are expressed in a series solution of the form

$$\left(W,\Theta\right)=\sum_{i=0}^{N}\left(a^2\right)^i\left(W_i,\Theta_i\right) \qquad (17)$$

Substituting Eq. (17) in to Eqs. (12)–(13) and collecting terms of zeroth –order we get

$$\left(1+G\right)D^2W_0=0 \qquad (18)$$

$$D^2\Theta_0-PeD\Theta_0=f(z)W_0. \qquad (19)$$

Then solutions to the above equations are

$$W_0=0\quad\text{and}\quad\Theta_0=1 \qquad (20)$$

The first–order equations are

$$\left(1+G\right)D^2W_1=-R(1+\lambda z) \qquad (21)$$

$$D^2\Theta_1 - PeD\Theta_1 = 1 + f(z)W_1. \tag{22}$$

The general solution of Eq. (21) is

$$W_1 = \frac{-R}{(1+G)}\left(\frac{z^2}{2} + \lambda\frac{z^3}{6}\right) + c_1 z + c_2 \tag{23}$$

From solvability condition, we have

$$\int_0^1 [1 + f(z)W_1]\,dz = 0 \tag{24}$$

The expressions for W_1 is substituted into Eq. (24) and integrated to yield an expression for the critical Rayleigh number R_c for different types of boundaries.

The boundary conditions are;

$$W_1 = 0\,, D\Theta = 0 \quad at \quad z = 0, 1. \tag{25}$$

Hence the values of c_1 and c_2 are

$$c_1 = \frac{R}{(1+G)}\left(\frac{1}{2} + \frac{\lambda}{6}\right),\ c_2 = 0.$$

Then expression for R_c is given by

$$R_c = \frac{12(1+G)Pe^4\left(e^{Pe}-1\right)}{\left[-\left(e^{Pe}-1\right)(3Ns+2\lambda)Pe^3 + 2e^{Pe}(2Ns+Pe) - 6\lambda(Pe-\lambda) - \lambda Pe\left(6e^{Pe}+(\lambda+3)Pe\right)\right]}. \tag{26}$$

In the absence of internal heating $\left(i.e\ \ Ns \to 0\right)$, the above expression reduces to

$$R_c = \frac{6(1+G)Pe^3}{\left[(3+2\lambda)Pe^2\coth\left(Pe/2\right) - 6(Pe-\lambda) - \lambda Pe\dfrac{\left(6e^{Pe}+Pe\right)}{\left(e^{Pe}-1\right)}\right]}. \tag{27}$$

In the absence of gravitational field $\left(i.e\,\lambda \to 0\right)$, the above expression reduces to

$$R_c = \frac{2(1+G)Pe^2}{\left[Pe\coth\left(Pe/2\right) - 2\right]} \tag{28}$$

and when Darcy equation is used to describe the porous medium $\left(i.e\ \ G \to 0\right)$, the above expression reduces to that of Nield [6].

$$R_c = \frac{2Pe^2}{\left[Pe\coth\left(Pe/2\right) - 2\right]}. \tag{29}$$

In the absence of throughflow $\left(i.e\,Pe \to 0\right)$, Eq. (29) reduces to $R_c = 12$, which is known exact value.

IV. Results and Discussion

The onset of penetrative convection via internal heating in the presence of a vertical through flow and variable gravity field is considered in a system consisting of a porous layer. In the calculation, we have chosen the value of $\phi = 0.389$, C_b =209.25and $\sqrt{Da} = 3.04 \times 10^{-3}$ which correspond to 3 cm deep porous layer consisting of 3mm diameter glass beads.

The critical Rayleigh number computed from Eqs. (26) and (33) in the absence of variable gravity field (i.e., $\lambda = 0$) and internal heating(i.e., $Ns = 0$) are compared with those obtained from Nield [6] in Table. 1 for two types of boundary conditions and the results are found to be in good agreement, when Darcy equation is used in the porous layer. When $Ns = 0$, $\lambda = 0$ and $Pe = 0$, the known exact value $R_c = 12$ (Nield [6]) is retrieved. The direction of throughflow has no influence on the stability of the system in the absence of internal heating and throughflow is to delay the onset of convection. This may be attributed to the fact that the primary effect of throughflow is to confine significant thermal gradients to a thermal boundary layer at the boundary toward which the throughflow is directed. The effective length scale is thus smaller than the layer thickness d and hence the effective Rayleigh number which will be depending on d^3 will be much less than the actual Rayleigh number. A larger value of Rayleigh number is thus necessary to initiate the convection. Nonetheless, the simultaneous presence of throughflow and internal heating alters the basic temperature distribution so that the direction of throughflow also affects the stability of the system. As a result of this some unusual behaviours are observed namely, (i) downward throughflow initially shows some destabilizing effect (Fig. 1) and (ii) increasing internal heat source strength causes stabilizing effect for upward throughflow initially (Fig.2).

Figures 3 and 4 exhibit plots of R_c as a function of Pe for two values of variable gravity parameter $\lambda = -1$, 1 respectively with three values of Prandtl number $Pr = 0.1$, 1 and 100. From Figs. 4 and 5 it is observed that for all values of Pr, both upward and downward throughflow stabilizes the system in the absence of internal heating($Ns = 0$). In the presence of internal heating ($Ns = 5$) upward throughflow stabilizes the system for all values of Pr while downward throughflow destabilizes the system when $-4.6 \le Pe \le 0, -5.2 \le Pe \le 0, -5.4 \le Pe \le 0$ $-2.4 \le Pe \le 0$, $-3.6 \le Pe \le 0$, $-3.7 \le Pe \le 0$ for Pr = 0.1,1 and 100, when $\lambda = -1, 1$ respectively, and for higher values of Pe the system is stabilizing. And also from Figs 3 and 4, it can be observed that, R_c is maximum when the variable gravity parameter λ takes the value -1. This is because, as noted by Chen and Chen [4] when $\lambda < 0$, the buoyancy effects are maximum at the lower wall and decrease with height. Further, the critical Rayleigh number is based on the gravity level at the lower wall. Thus when $\lambda < 0$, the gravity level in the layer is decreased, which thereby causes an increase in the critical Rayleigh number. When λ is positive the opposite effect is seen. That is, the critical Rayleigh number is less than that for the constant gravity case $\lambda = 0$.

Table 1**:** Comparison of critical Rayleigh number with those of Nield [6] with $(\lambda = 0, Ns = 0 \text{ and } G = 0)$

Pe	Nield [6]	Present study
0	12	12
1.0	12.1986	12.1885
2.0	12.7781	12.7683
3.0	13.6947	13.6747
4.0	14.8889	14.8784
5.0	16.2981	16.2882

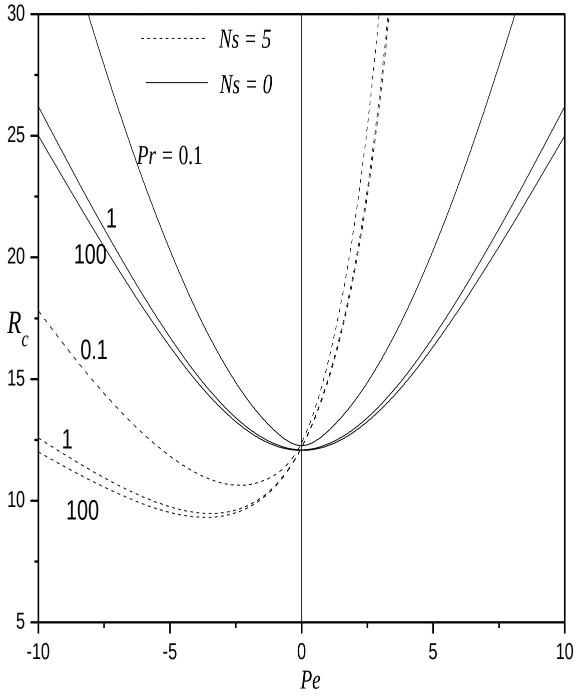

Fig. 1: Variation of critical Rayleigh number versus Pe for different values of Pr and Ns with **λ**=0

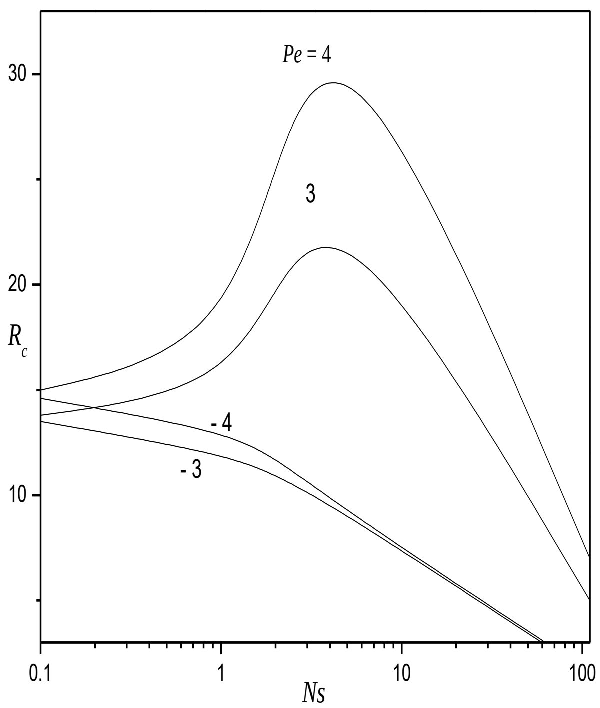

Fig. 2: Variation of critical Rayleigh number versus Ns for different values of Pe with Pr = 10, **λ**=0

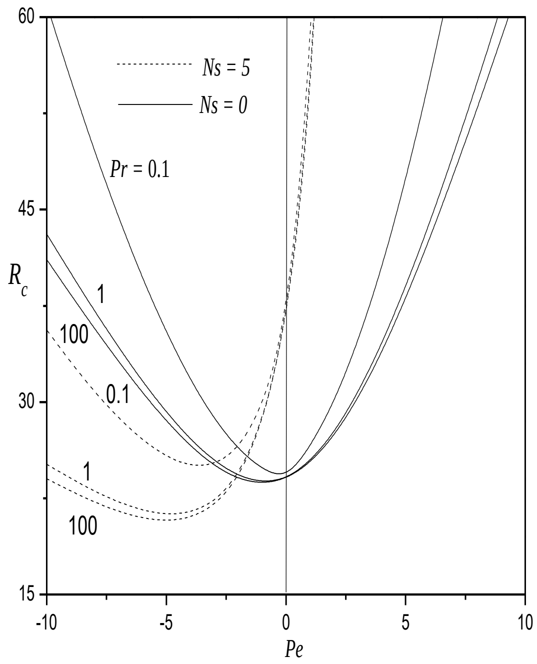

Fig. 3: Variation Critical Rayleigh number versus Pe for different values of Pr and Ns with **λ**=-1

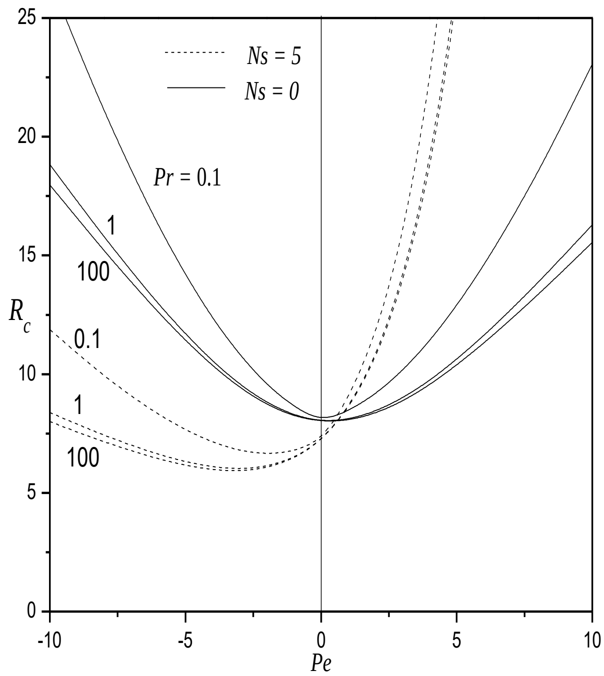

Fig. 4: Variation Critical Rayleigh number versus Pe for different values of Pr and Ns with **λ**=1

V. Conclusions

The onset of penetrative convection via internal heating in porous layer is studied in the presence of a vertical throughflow and variable gravity field. From the foregoing analysis, it is observed that the stability characteristics of the configuration depend crucially on (i) the presence of internal heating in the porous layer, (ii) variable gravity field, (iii) types of boundary conditions and (iv) throughflow direction. In absence of both gravity field and internal heating some usual behaviours are observed namely, (i) when the lower and upper boundaries are of same type, the critical Rayleigh number is an even function of Pe and system gets stabilized irrespective of direction of throughflow (ii) when the two boundaries are different types throughflow in one direction will be destabilize the system up to certain values of Pe .

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