On Contra Alpha Weakly Semi Continuous Function In Topological Spaces

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Abstract –In this paper, we introduce some new class of functions called contra αws-continuous functions by using αws-closed sets. Also we established the relationship between contra αws-continuous function with some other contra continuous functions. Also we define contra αws-irresolute function, perfectly contra αws-irresolute functions, almost contra αws-continuous functions.

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***Key words**:* contra αws-continuous function, contra αws-irresolute function, perfectly contra αws-irresolute functions, almost contra αws-continuous functions.

1. INTRODUCTION

In 1996,J.Dontchev [3] introduced the notion of contra continuity. In 1999, S. Jafari and T. Noiri [12] introduced a new class of mapping called contra α-continuous functions. In this paper, the authors introduced a new class of functions, namely contra αws-continuous functions,contraαws-irresolute function, perfectly contra αws-irresolute functions, almost contra αws-continuous functions in topological spaces. Section 2 contains the certain preliminary concepts, Section 3 contain the concept of contra αws-continuous function, perfectly contraαws-irresolute, almost contraαws-continuous and contra αws-irresolutefunctions are studied and the diagram also included which states the relationships among the generalized contra continuous functions in topological spaces and Section 4 contains conclusions and Section 5 contains references.

2. PRELIMINARIES

Throughout this paper (X,τ), (Y,σ) and (Z,μ) represents the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space (X,τ), *cl*A and *int*A denote the closure of A and the interior of A respectively. X – A denotes the complement of A in X. We recall the following definitions.

Definition 2.1: A subset A of a space X is called

(i) α-open [10] if A ⊆*int cl int*A and α-closed if *cl intcl*A⊆ A.

(ii) π-open [18] if A is the union of regular open sets and π-closed if A is the intersection of regular closed sets.

The alpha-closure of a subset A of X is the intersection of all alpha-closed sets containing A and is denoted by αclA.

Definition 2.2: A subset A of a space X is called

(i) generalized closed [7] (briefly g-closed) if *cl*A⊆ U whenever A ⊆ U and U is open.

(ii)regular generalized closed [11] (briefly rg-closed) if *cl*A⊆ U whenever A ⊆ U and U is regular open.

(iii) generalized semi-closed [2] (briefly gs-closed) if *scl*A⊆ U whenever A ⊆ U and U is open.
(iv)generalized pre-closed [8] (briefly gp-closed) if *pcl*A⊆ U whenever A ⊆ U and U is open.
(v) generalized regular closed [13] (briefly gr-closed) if *rcl*A⊆ U whenever A ⊆ U and U is open.(vi) π -generalized closed [4] (briefly πg -closed) if *cl*A⊆ U whenever A ⊆ U and U isπ-open.
(vii) generalized b-closed [1] (briefly gb-closed) if *bcl*A⊆ U whenever A ⊆ U and U is open.
(viii) Regular generalized b-closed [9] (briefly rgb-closed) if *bcl*A⊆ U whenever A ⊆ Uand U is regular open.
(ix) genralized pre regular-closed [6] (briefly gpr-closed) if *pcl*A⊆ U whenever A ⊆ U and U is regular-open.
(x) *g-closed [17] if *scl*A⊆ U whenever A ⊆ U and U is semi-open.

The complements of the above mentioned closed sets are their respective open sets. For example a subset B of a space X is generalized open (briefly g-open) if X – B is g-closed.

Definition 2.3: [15]A subset A of a space X is called Alpha Weakly Semi closed (brieflyαws-closed) if *αcl*A ⊆ U whenever A ⊆ U and U is ws-open.

Definition 2.4: A function f: (X,τ)→ (Y,σ) is called
ii) αws-continuous function if f^{-1}(V) is αws-closed in (X,τ) for every closed subset V in (Y,σ).
iv) αws-irresolute if f^{-1}(V) is αws-closed in (X,τ) for every αws-closed subset V in (Y,σ).

Definition 2.5: A function f: (X,τ)→ (Y,σ) is called
i) contrag-continuous if f^{-1}(V) is g-closed in (X,τ) for every open subset V of (Y,σ) [14].
ii)contraα-continuous if f^{-1}(V) is α-closed in (X,τ) for every open subset V of (Y,σ) [12].
iii) contracontinuous if f^{-1}(V) is closed in (X,τ) for every opensubset V of (Y,σ)[3].
iv) contraπ-continuous if f^{-1}(V) is π-closed in (X,τ) for every open subset V of (Y,σ) [5].
We use the following notations.
αWSC(X,τ) – The collection of all αws-closed sets in (X,τ)
αWSO(X,τ) – The collection of all αws-open sets in (X,τ)

3. Contraαws-continuous functions

Definition3.1: Afunction f: (X,τ)→ (Y,σ) is called contraαws-continuous function if f^{-1}(V) is αws-closed in (X,τ) for every open subset V of (Y,σ).

Preposition 3.2: Letf: (X,τ)→ (Y,σ) be a function
i) If f is acontra-continuous function, then f is contraαws-continuous function.
ii) If f is contra α-continuous function, then f is contraαws-continuous function.
iii) If f is contra π-continuous function, then f is contra αws-continuous function.
Proof:
Assume that the functionf: (X,τ)→ (Y,σ) is contra-continuous function(resp.Contraα-continuous function, contra π-continuous function). Let us assume V be an open subset of (Y,σ). Since f is a contra-continuous function (resp. contra α-continuous function, contra π-continuous function), f^{-1}(V) is closed (resp. α-closed,π-closed) in (X,τ). By Preposition 3.2(i), (ii), (iii) [15],f^{-1}(V) is αws-closed in (X,τ). Hence f is contraαws-continuous function. This proves (i) and (ii) and (iii).

The reverse Implication of the above preposition need not be true as shown in the following Example.

Example 3.3: Let X= {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Let Y= {a, b, c} with σ= {ϕ, {a}, Y}.Let the function f: (X,τ)$\rightarrow$ (Y,σ) be defined as f(a) = c; f(b) = a; f(c) = b. Then f is contra αws-continuous function, but not contra continuous function, not contraα-continuous function, not contraπ-continuous function.

Preposition 3.4: Letf: (X,τ)$\rightarrow$ (Y,σ) be a function if f is contraαws-continuous function, then f is contra gs-continuous function, contra gb-continuous function, contrargb-continuous functioncontra gp-continuous function, contra gpr-continuous function.

Proof:

Suppose f is contraαws-continuous function. Let us assume V be an open subset of (Y,σ). Since f is contra αws-continuous function, f^{-1}(V) isαws-closed in (X,τ). By preposition 3.7 [15],f^{-1}(V) is gs-closed in (X,τ). Thus f iscontrags-continuous function.

Similarly we can prove f is contra gb-continuous, contrargb-continuous, contra gp-continuous, contra gpr-continuous by Preposition 3.7 [15].

The reverse Implication of the above preposition need not be true as shown in the Example 3.5.

Example 3.5: Let X= Y ={a, b, c, d} with τ= {ϕ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, X}with σ = {ϕ, {a, c}, Y}.Let the function f: (X,τ)$\rightarrow$ (Y,σ) be defined as f(a) = b; f(b) = a; f(c) = b; f(d) = c. Then f is contra gs-continuous function, contra rgb-continuous function, contragb-continuous function, contragp-continuous function, contra gpr-continuous function but not contra αws-continuous function.

The following examples shows that the concept of contraαws-continuous function is independent fromthe concept of contra g-continuous, contra rg-continuous, contra *g-continuous, contra πg-continuous, contra gr-continuousfunction.

Example 3.6: From Example3.5, f^{-1}({a, c}) = {b, d}, is g-closed, rg-closed, *g-closed, πg-closed, gr-closed in (X,τ). Therefore, f is contra g-continuous, contra rg-continuous, contra *g-continuous, contra πg-continuous, contra gr-continuousfunction. But {b, d} is not αws-closed in (X,τ). Hence f is not a contra αws-continuous functions.

Example 3.7: Let X= Y ={a, b, c, d} with τ= {ϕ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, X} withσ = {ϕ, {b}, Y}.Let the function f: (X,τ)$\rightarrow$ (Y,σ) be defined as f(a) = b; f(b) = a; f(c) = b; f(d) = c. Now f^{-1}({b}) = {c}, which is αws-closed but not g-closed, not rg-closed, not *g-closed, notπg-closed, not gr-closed in (X,τ). Hence f is contra αws- continuous function but not contra g-continuous, not contra rg-continuous, not contra *g-continuous, not contra πg-continuous, not contra gr-continuous functions.

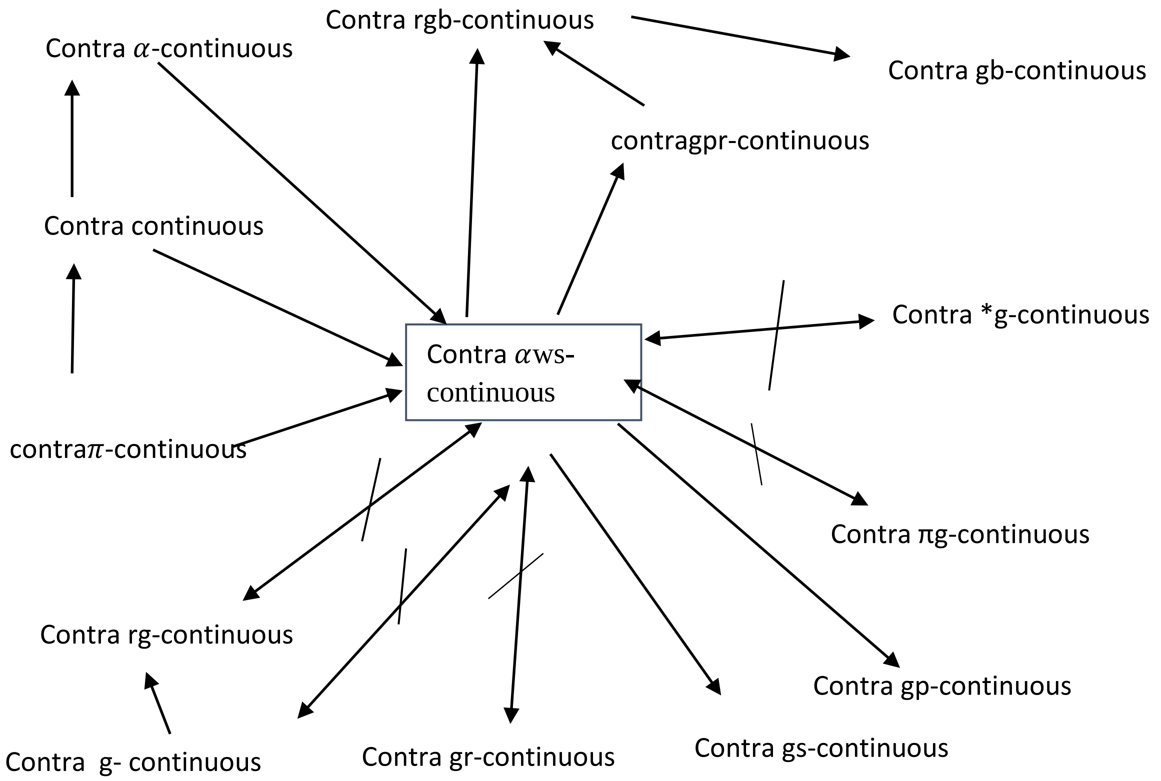

Figure: 1

Definition 3.8: A function f: (X, τ)→(Y, σ) is called Contra αws-irresolute if $f^{-1}(V)$ is αws-closed in (X,τ) for every αws-open subset V of (Y,σ).

The following examples shows that the concept of αws-irresolute function and contra αws-irresolute are independent of each other.

Example 3.9: Let X=Y = {a, b, c} with τ= {ϕ, {b, c}, X} and σ = {ϕ, {a}, {a, c}, Y}.Let the function f: (X,τ)→ (Y,σ) be defined as f(a) = a; f(b) = b; f(c) = c.ThenαWSC(X,τ) = {ϕ, {a}, {a, b}, {a, c}, X}, αWSO(Y,τ) = {ϕ, {a}, {a, c}, {a, b}, Y} and αWSC(Y,τ) = {ϕ, {b}, {c}, {b, c}, Y}. Now f^{-1}({a}) ={a}, which is αws-closed in (X,τ). Then f is Contra αws-irresolute. Also f^{-1}({b}) = {b}, which is not αws-closed in (X,τ). Then f is not αws-irresolute functions.

Example 3.10: Let X=Y = {a, b, c} with τ= {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {b, c}, Y}.Let the function f: (X,τ)→ (Y,σ) be defined as f(a) = b; f(b) = c; f(c) = a. Then αWSC(X,τ) = {ϕ, {c}, {b, c}, {a, c}, X}, αWSO(Y,τ) = {ϕ, {b}, { c}, {b, c}, Y} and αWSC(Y,τ) = {ϕ, {a}, {a, b}, {a, c}, Y}. Now f^{-1}({a}) = {c}, which is αws-closed in (X,τ). Then f isαws-irresolute. Also f^{-1}({c}) = {b}, which is not αws-closed in (X,τ). Then f is notcontra αws-irresolute functions.

Theorem 3.11:
Every contraαws-irresolute function is contra αws-continuous function.

Proof:

Let us assume f: $(X, \tau)\rightarrow(Y, \sigma)$ be a contra αws-irresolute function. Let us assume V be a open subset of (Y,σ). Since every open set is an α-open and [16], V is αws-open in (Y,σ). Since f is contra αws-irresolute function,$f^{-1}(V)$ isαws-closed in (X,τ).Hence f is contra αws-continuous.

The converse of the above theorem is not true us shown in the example 3.12.

Example 3.12:
Let X=Y= {a, b, c} with $\tau = \{\phi, \{a\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{c\}, \{b, c\}, Y\}$. Let us assume f: $(X, \tau) \rightarrow(Y, \sigma)$ be defined as f(a) = a, f(b) = b and f(c) = c. Then $\alpha WSC(X,\tau) = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$, $\alpha WSO(Y,\tau) = \{\phi, \{c\}, \{ b, c\}, \{a, c\}, Y\}$. Here the inverse image of αws-open set {a, c} in (Y,σ) are {a, c} which is not αws-closed in (X,τ). Hence f is contra αws-continuous function but not contra αws-irresolute function.

Theorem 3.13:
Letf: $(X, \tau)\rightarrow(Y, \sigma)$ and g: $(Y, \sigma)\rightarrow(Z,\mu)$ be two functions and let $h=g\circ f$. Then
i. h is contra αws-continuous function if f is contra αws-continuous function & g is continuous.
ii. h is contraαws-continuous function if f is αws-irresolute function and g is contra continuous.
iii) h is contraαws-continuous function if f is αws-irresolute function and g is contra αws-continuous.
iv) h is αws-continuous function and contra αws-continuous function if f is contra αws-continuous function and g is perfectly continuous.
v) h is αws-continuous function if f is contra αws-continuous function and g is contra continuous.

Proof:
i) Suppose V is an open in (Z,μ). Since g is continuous,$g^{-1}(V)$ is open in (Y,σ).Since f is contra αws-continuous,$f^{-1}(g^{-1}(V))$ is αws-closed in (X,τ).Thus $(g\circ f)^{-1}(V)$is αws-closed in (X,τ). Therefore $h^{-1}(V)$ αws-closed in (X,τ). Hence h is contraαws-continuous.

ii) Suppose V is an open in (Z,μ). Since g is contra continuous, $g^{-1}(V)$ is closed in (Y,σ). By preposition 3.2 [15], $g^{-1}(V)$ is αws-closed in (Y,σ). Since f is αws-irresolute, $f^{-1}(g^{-1}(V))$ is αws-closed in (X,τ). Thus $(g\circ f)^{-1}(V)$is αws-closed in (X,τ). Therefore $h^{-1}(V)$ αws-closed in (X,τ). Hence h is contraαws-continuous.

iii) Suppose V is an open in (Z,μ). Since g is contra αws-continuous function, $g^{-1}(V)$ is αws-closed in (Y,σ). Since f is αws-irresolute, $f^{-1}(g^{-1}(V))$ is αws-closed in (X,τ). Thus $(g\circ f)^{-1}(V)$is αws-closed in (X,τ). Therefore $h^{-1}(V)$ αws-closed in (X,τ). Hence h is contraαws-continuous.

iv) Suppose V is closed in (Z,μ). Since g is perfectly continuous, $g^{-1}(V)$ is clopen in (Y,σ). Since f is contra αws-continuous,$f^{-1}(g^{-1}(V))$ is bothαws-closed and αws-open in (X,τ).Thus $(g\circ f)^{-1}(V)$is bothαws-closed and αws-open in (X,τ). Hence h isαws-continuous and contraαws-continuous.

v) Suppose V is closed in (Z,μ). Since g is contra continuous, $g^{-1}(V)$ is open in (Y,σ). Since f is contra αws-continuous, $f^{-1}(g^{-1}(V))$ is αws-closed in (X,τ). Thus $(g\circ f)^{-1}(V)$is αws-closed in (X,τ). Therefore $h^{-1}(V)$ αws-closed in (X,τ). Hence h is αws-continuous.

Theorem 3.14:

Let us assume f: (X, τ)→(Y, σ) be a function. Then the following two statements are equivalent

i) f is contraαws-continuous function

ii) The inverse image of each closed set in (Y,σ) is αws-open in (X,τ).

Proof:

(i) ⇒ (ii) Assume (i) holds. Let us assume V be a closed in (Y,σ). Then Y – V is open in (Y,σ).By our assumption, f^{-1}(Y–V)isαws-closed in (X,τ). But f^{-1}(Y – V) = X –f^{-1}(V) whichis αws-closed in (X, τ). That is, f^{-1}(V) is αws-open in (X, τ). This proves (i) ⇒ (ii)

(ii) ⇒ (i)

Let us assume V be an open in (Y,σ). Then Y – V is closed in (Y,σ). By our assumption, f^{-1}(Y–V)isαws-open in (X,τ). But f^{-1}(Y – V) = X –f^{-1}(V) whichis αws-open in (X, τ). That is, f^{-1}(V) is αws-closed in (X, τ). This proves (ii) ⇒ (i)

Definition 3.15:

A function f: (X, τ) →(Y, σ) is called perfectly contra αws-irresolute function if f^{-1}(V) is both αws-closed and αws-open in (X, τ) for every αws-open subset V of (Y,σ)

Theorem 3.16: A function f is perfectly contra αws-irresolute iff f is contra αws-irresolute and αws-irresolute.

Proof:

Suppose f is perfectly contraαws-irresolute. To prove f is contra αws-irresolute and αws-irresolute. Let V be the αws-open set of (Y, σ). Since f is perfectly contraαws-irresolute,f^{-1}(V) bothαws-closed and αws-openin (X, τ). Hence f is contra αws–irresolute and αws-irresolute. Conversely, suppose f iscontra αws–irresolute and αws-irresolute. Let V be αws-open in (Y, σ). Since f is contra αws–irresolute and αws-irresolute, f^{-1}(V) is both αws-closed and αws-open in (X, τ). Hence f is perfectly contra αws-irresolute..

Definition 3.17:

A function f: (X, τ) → (Y, σ) is called almost contra αws-continuous function if f^{-1}(V) is αws-closed in (X, τ) for every regular open subset V of (Y, σ).

Theorem 3.18:

Every contraαws-continuous function is almost contra αws-continuous function.

Proof:

Let f: (X, τ) → (Y, σ) be a function. Suppose f is contra αws-continuous. Let V be a regular open subset of (Y, σ). Since every regular open set is open, V is open in (Y, σ). Since f is contra αws-continuous function, f^{-1}(V) is aαws-closed in (X, τ). Hence f isalmostαws-continuous function.

Theorem 3.19:

Let f: (X, τ) → (Y, σ) be a function Then the following two statements are equivalent

i) f is almost contraαws-continuous function

ii) The inverse image of each closed set in (Y,σ) is αws-open in (X,τ).

Proof:

(i) $\Rightarrow$ (ii) Assume (i) holds. Let us assume V be a regular closed in (Y,σ). Then Y – V is regular open in (Y,σ). By our assumption, f^{-1}(Y–V) is αws-closed in (X,τ). But $f^{-1}(Y - V) = X - f^{-1}(V)$ whichis αws-closed in (X, τ). That is, $f^{-1}(V)$ is αws-open in (X, τ). This proves (i) $\Rightarrow$ (ii)

(ii) $\Rightarrow$ (i) Let us assume V be a regular open in (Y,σ). Then Y – V is regular closed in (Y,σ). By our assumption, f^{-1}(Y–V) is αws-open in (X,τ). But $f^{-1}(Y - V) = X - f^{-1}(V)$ whichis αws-open in (X, τ). That is, $f^{-1}(V)$ is αws-closed in (X, τ). This proves (ii) $\Rightarrow$ (i)

4. Conclusion

In this paper, we introduced contraαws-continuous function, perfectly contraαws-irresolute, almost contraαws-continuous,contraαws-irresolute function and characterized with analogous recent concepts in the literature of general topology.

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