Various distances on $q^\text{th}$-rung orthopair picture fuzzy set and MABAC method for multiple attribute group decision making

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Abstract— The aim of this paper to introduce the new concept of $q^\text{th}$-rung orthopair picture fuzzy set(q*P) and to find some of the distances using the formulas and to find the application of $q^\text{th}$-ROPFS using MABAC for multiple attribute group decision making method.

Keywords—$q^\text{th}$-rung orthopair picture fuzzy set, Hamming distance, Euclidean distance, Bored approximation area (BAA).

I. INTRODUCTION

Fuzzy set theory developed by Zadeh in 1965 which plays an important role to take a decision in uncertain situations. In 1983, Atanassov introduced intuitionistic fuzzy set as a generalization of fuzzy set theory, where each element has a degree of membership and degree of non-membership values. In 2012, A.A.Salama and S.A.Ablawi introduced the concepts of neutrosophic topological spaces besides the degree of membership, degree of indeterminacy and the degree of non-membership for each element. In 2013 Yager introduced the Pythagorean fuzzy set as an extension of intuitionistic fuzzy set. In 2017 he introduced $q^\text{th}$-rung orthopair fuzzy set as a new generalization of orthopair fuzzy sets, which relax the constraint of orthopair membership grades with $\mu^q_1+\nu^q_2\leq 1, (q\geq 1)$, then Cuong and Kreinovich introduced picture fuzzy set theory and now we extend it to $q^\text{th}$-rung orthopair picture fuzzy set with the condition $(\mu^q_1+\eta^q_3+\nu^q_4)\leq 1, (q\geq 1)$.

II. BASIC DEFINITIONS

Definition 2.1[1]
Let $X$ be a universe of discourse. An IFS in $X$ is given by $I=\{<x,\mu(x),\nu(x)>:x\in X\}$ where $\mu: X\rightarrow [0,1]$ denotes the degree of membership $\nu: X\rightarrow [0,1]$ denotes the degree of non-membership of the element $x\in X$ to the set $I$ with the condition that $0\leq \mu(x)+\nu(x)\leq 1$, and the degree of indeterminacy $\pi(x)=1-\mu(x)-\nu(x)$.

Definition 2.2[7]
Let $X$ be a universe of discourse. A Pythagorean fuzzy set(PFS) $P$ in $X$ is given by $P=\{<x,\mu(x),\nu(x)>:x\in X\}$ where $\mu: X\rightarrow [0,1]$ denotes the degree of membership $\nu: X\rightarrow [0,1]$ denotes the degree of non-membership of the element $x\in X$ to the set $P$ with the condition that $0\leq (\mu(x))^2+(\nu(x))^2\leq 1$, and the degree of indeterminacy $\pi(x)=(1-\mu(x)-\nu(x))^2$.

Definition 2.3[8]
Let $X$ be a universe of discourse. A $q^\text{th}$-rung orthopair fuzzy set $A$ in $X$ is given by $A=\{<x,\mu(x),\nu(x),\eta(x)>:x\in X\}$ where $\mu: X\rightarrow [0,1]$ denotes the degree of membership $\nu: X\rightarrow [0,1]$ denotes the degree of non-membership of the element $x\in X$ to the set $A$ with the condition that $0\leq (\mu(x))^q+(\nu(x))^q\leq 1$, and the degree of indeterminacy $\pi(x)=(1-(\mu(x))^q-(\nu(x))^q)^{1/q}$.

Definition 2.4[4]
A Picture Fuzzy Set(PFS) $A$ on a universe $X$ is of the form $A=\{<x,\mu_A(x),\nu_A(x)>:x\in X\}$ where $\mu_A: X\rightarrow [0,1]$ denotes the degree of positive membership, $\eta_A: X\rightarrow [0,1]$ denotes the degree of neutral membership, $\nu_A: X\rightarrow [0,1]$ denotes the degree of negative membership of the element $x\in X$ to the set $A$ with the condition that $0\leq (\mu_A(x))+ (\eta_A(x))+ (\nu_A(x))\leq 1$, then $1-(\mu_A(x)+\eta_A(x)+\nu_A(x))$ is called the degree of refusal of membership of $x$ in $A$. 

http://infokara.com/
Definition 2.5
A q-th rung orthopair picture fuzzy set A on a universe X is of the form $A = \{x, \mu_A(x), \eta_A(x), \nu_A(x) : x \in X\}$ where $\mu_A : X \to [0,1]$ denotes the degree of positive membership, $\eta_A : X \to [0,1]$ denotes the degree of neutral membership, $\nu_A : X \to [0,1]$ denotes the degree of negative membership of the element $x \in X$ to the set $A$ with the condition that $0 \leq (\mu_A(x))^q + (\eta_A(x))^q + (\nu_A(x))^q \leq 1$, then $(1-(\mu_A(x))^q + (\eta_A(x))^q + (\nu_A(x))^q)^{\frac{1}{q}}$ is called the degree of refusal of membership of $X$ in $A$.

III. VARIOUS DISTANCE MEASURES

Definition 2.6
Let $P, Q, R \in \mathbb{Q}^q$-ROPFS on $X$. The distance measure, $d$ is a mapping $d : \mathbb{Q}^q$-ROPFS x $\mathbb{Q}^q$-ROPFS $\to [0,1]$ satisfying the following properties,

(i) $0 \leq D(P, Q) \leq 1$
(ii) $D(P, Q) = D(Q, P)$
(iii) $D(P, Q) = 0$ if and only if $P = Q$
(iv) $D(P, Q) \leq D(P, R) + D(R, Q)$

Definition 2.7
(i) Hamming Distance: $d^{hi}_{\mathbb{Q}^q}$-ROPFS($A, B$) =

$$
\frac{1}{2n} \sum_{x \in X} \left| \sum_{i=1}^{n} \left( |\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)| \right) \right|
$$

(ii) Normalized Hamming Distance: $d^{hi}_{\mathbb{Q}^q}$-ROPFS($A, B$) =

$$
\frac{1}{2n} \sum_{x \in X} \left| \sum_{i=1}^{n} \left( |\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)| \right) \right|
$$

(iii) Euclidean Distance: $d^{E}_{\mathbb{Q}^q}$-ROPFS($A, B$) =

$$
\sqrt{\frac{1}{2n} \sum_{x \in X} \left| \sum_{i=1}^{n} \left( |\mu_A(x_i) - \mu_B(x_i)|^2 + |\eta_A(x_i) - \eta_B(x_i)|^2 + |\nu_A(x_i) - \nu_B(x_i)|^2 + |\pi_A(x_i) - \pi_B(x_i)|^2 \right) \right|}
$$

(iv) Normalized Euclidean Distance: $d^{NE}_{\mathbb{Q}^q}$-ROPFS($A, B$) =

$$
\sqrt{\frac{1}{2n} \sum_{x \in X} \left| \sum_{i=1}^{n} \left( |\mu_A(x_i) - \mu_B(x_i)|^2 + |\eta_A(x_i) - \eta_B(x_i)|^2 + |\nu_A(x_i) - \nu_B(x_i)|^2 + |\pi_A(x_i) - \pi_B(x_i)|^2 \right) \right|}
$$

IV. NUMERICAL EXAMPLE

Let $A = \{0.3, 0.4, 0.5\}$ and $B = \{0.4, 0.5, 0.6\}$ when $n=4$

$\pi_A(x_1) = \frac{1}{4}(1 - (0.4^4 + 0.3^4 + 0.5^4)) = \frac{1}{4}(0.9407) = 0.2352
\pi_A(x_2) = \frac{1}{4}(1 - (0.3^4 + 0.4^4 + 0.5^4)) = \frac{1}{4}(0.9038) = 0.2259
\pi_A(x_3) = \frac{1}{4}(1 - (0.4^4 + 0.3^4 + 0.5^4)) = \frac{1}{4}(0.9038) = 0.2259
\pi_B(x_1) = \frac{1}{4}(1 - (0.5^4 + 0.4^4 + 0.7^4)) = \frac{1}{4}(0.6718) = 0.1679
\pi_B(x_2) = \frac{1}{4}(1 - (0.4^4 + 0.6^4 + 0.8^4)) = \frac{1}{4}(0.4352) = 0.1088
\pi_B(x_3) = \frac{1}{4}(1 - (0.6^4 + 0.7^4 + 0.8^4)) = \frac{1}{4}(0.2207) = 0.0552

(i) Hamming Distance: $d^{hi}_{\mathbb{Q}^q}$-ROPFS($A, B$) =

$$
\frac{1}{2n} \sum_{x \in X} \left| \sum_{i=1}^{n} \left( |\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)| \right) \right|
$$
\[
\begin{align*}
&= \frac{1}{2(3)} \{0.0369 + 0.0175 + 0.2145 + 0.104 + 0.3471 + 0.104 + 0.232 + 0.3471 + 0.0795 + 0.1628 + 0.2896\} \\
&= \frac{1}{6}[1.9525] = 0.3254 \\
(ii) \text{ Normalized Hamming Distance: } d_{\text{RORFS}}^N(A,B) = \\
&= \frac{1}{2(n \cdot (3)(3))} \{0.0369 + 0.0175 + 0.2145 + 0.104 + 0.3471 + 0.104 + 0.232 + 0.3471 + 0.0795 + 0.1628 + 0.2896\} \\
&= \frac{1}{18}[1.9525] = 0.11 \\
(iii) \text{ Euclidean Distance: } d_{\text{RORFS}}^E(A,B) = \\
&= \sqrt{\frac{1}{2(n \cdot (3))} \{[\mu_A^q(x_i) - \mu_B^q(x_i)]^2 + [\eta_A^q(x_i) - \eta_B^q(x_i)]^2 + [v_A^q(x_i) - v_B^q(x_i)]^2 + [\pi_A^q(x_i) - \pi_B^q(x_i)]^2\}} \\
&= \sqrt{\frac{1}{2(3)} [0.0369^2 + 0.0175^2 + 0.2145^2 + 0.104^2 + 0.3471^2 + 0.104^2 + 0.232^2 \\
&+ 0.3471^2 + 0.0795^2 + 0.1628^2 + 0.2896^2]} \\
&= \frac{1}{6}[0.4811] = 0.2832 \\
(iv) \text{ Normalized Euclidean Distance: } d_{\text{RORFS}}^{NE}(A,B) = \\
&= \sqrt{\frac{1}{2n \cdot (3)} \{[\mu_A^q(x_i) - \mu_B^q(x_i)]^2 + [\eta_A^q(x_i) - \eta_B^q(x_i)]^2 + [v_A^q(x_i) - v_B^q(x_i)]^2 + [\pi_A^q(x_i) - \pi_B^q(x_i)]^2\}} \\
&= \sqrt{\frac{1}{2(3)(3)} [0.0369^2 + 0.0175^2 + 0.2145^2 + 0.104^2 + 0.3471^2 + 0.104^2 + 0.232^2 \\
&+ 0.3471^2 + 0.0795^2 + 0.1628^2 + 0.2896^2]} \\
&= \frac{1}{18}[0.4811] = 0.1414 = 0.1 \\
\end{align*}
\]

From the above values we obtain the following inequalities:

\[0.1 \leq 0.11 \leq 0.2832 \leq 0.3254\]

(i.e) N.E \leq N.H \leq E \leq H.
V. MABAC METHOD FOR MULTIPLE ATTRIBUTE GROUP

As the generalization of Intuitionistic Fuzzy Set (IFS) and Pythagorean Fuzzy Set (PFS), they are applied in many fields of decision making problems. There are some situations that IFS and PFS cannot be dealt with. For example, when the picture fuzzy experts are shown as (0.6, 0.7, 0.8) which indicates positive membership degree is 0.6, neutral membership is 0.7 and the negative membership is 0.8. In this case IFS and PFS cannot satisfy this situation. As the extension of fuzzy sets, q-ROPFS is more effective and scientific to solve more complex and uncertain problems. The Multi-Attribute Border Approximation area (MABAC) model controls the unknown decision making problems by calculating the distance between each alternatives and Bored Approximation Area (BAA). Based on that, here we choose q=4, then we can build the 4th-rung orthopair picture fuzzy set (4th-ROPFS) with MABAC model, where all the evaluation information are depicted by q-ROPFNs. Suppose there are m alternatives \((A_1, A_2, \ldots, A_m)\) and n attributes \((G_1, G_2, \ldots, G_n)\) with weighting vector \(w_j (j=1,2,\ldots,n)\) and \(\lambda\) experts \((d_1, d_2, \ldots, d_{\lambda})\) with weighting vector \((a_0, a_2, \ldots, a_{n_0})\), obtains the q-rung orthopair picture fuzzy evaluation matrix \(R=[A_{ij}]_{max}=(\mu_{ij}, \eta_{ij}, \nu_{ij}), i=1,2,\ldots,m\) and \(j=1,2,\ldots,n\) then \(\mu_{ij} \in [0,1]\) indicates the positive membership degree, \(\eta_{ij} \in [0,1]\) indicates the negative degree, \(\nu_{ij} \in [0,1]\) indicates the degree of non-membership degree.

**Definition 2.8[3]**

Let \(A_j=(\mu_{j}, \eta_{j}, \nu_{j})\) where \(j=1,2,\ldots,n\) are the list of q-ROPFS then the q-ROPFWA operator and q-ROPFWG can be expressed as follows:

\[
q^{th}\text{-ROPFWA} (a_1, a_2, a_3, \ldots, a_n) = \sum_{j=1}^{n} w_j a_j = \left(1 - \prod_{j=1}^{n} \left(1 - (\mu_{a_j})^q \right)^{w_j} \right), \prod_{j=1}^{n} \left(\eta_{a_j} \right)^{w_j}, \prod_{j=1}^{n} \left(\nu_{a_j} \right)^{w_j}, \text{ and}
\]

\[
q^{th}\text{-ROPFWG} (a_1, a_2, a_3, \ldots, a_n) = \prod_{j=1}^{n} \left(\mu_{a_j} \right)^{w_j}, 1 - \prod_{j=1}^{n} \left(1 - (\mu_{a_j})^q \right)^{w_j}, 1 - \prod_{j=1}^{n} \left(1 - (\eta_{a_j})^q \right)^{w_j}, \prod_{j=1}^{n} \left(\nu_{a_j} \right)^{w_j}.
\]

**Definition 2.9[3]**

Let \(A\) and \(B\) are the two q-ROPFN then we can obtain q-rung orthopair picture fuzzy Normalized Euclidean distance can be, \(d^{\text{NE-4th-ROPFS}}(A,B) = \sqrt{\frac{1}{2n} \sum_{i=1}^{n} \left[ (\mu_A^q (x_i) - \mu_B^q (x_i))^2 + (\eta_A^q (x_i) - \eta_B^q (x_i))^2 + (\nu_A^q (x_i) - \nu_B^q (x_i))^2 + (\pi_A^q (x_i) - \pi_B^q (x_i))^2 \right]}
\]

The decision-making steps of qth-rung orthopair fuzzy MABAC model can be expressed as follows.

Step 1: Establish the q-ROPFS evaluation matrix \(R=[A_{ij}]_{max}=(\mu_{ij}, \eta_{ij}, \nu_{ij})\) as below.

\[
R = \left[ A_{ij} \right]_{max} = \begin{bmatrix}
    (\mu_{11}, \eta_{11}, \nu_{11}) & (\mu_{12}, \eta_{12}, \nu_{12}) & \cdots & (\mu_{1n}, \eta_{1n}, \nu_{1n}) \\
    (\mu_{21}, \eta_{21}, \nu_{21}) & (\mu_{22}, \eta_{22}, \nu_{22}) & \cdots & (\mu_{2n}, \eta_{2n}, \nu_{2n}) \\
    \vdots & \vdots & \ddots & \vdots \\
    (\mu_{m1}, \eta_{m1}, \nu_{m1}) & (\mu_{m2}, \eta_{m2}, \nu_{m2}) & \cdots & (\mu_{mn}, \eta_{mn}, \nu_{mn})
\end{bmatrix}
\]

Where \([A_{ij}]_{max}=(\mu_{ij}, \eta_{ij}, \nu_{ij}), i=1,2,\ldots,m\) and \(j=1,2,\ldots,n\) q-ROPF information of alternative \(A_i (i=1,2,\ldots,m)\) on attribute \(G_j (j=1,2,\ldots,n)\) by experts \(d_j\).

Step 2: According to the q-ROPFWA or q-ROPWG aggregation operators, we utilize overall \([A_{ij}]\) to \(A_{ij}\), then the fused q-ROPFN matrix \(r=[A_{ij}]_{max}\) shown as follows:
Where $[A_{ij}]_{max}=(\mu_{ij}, \eta_{ij}, v_{ij})$, $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$ q-ROPF information of alternative $A_i$ ($i=1,2,\ldots,m$) on attribute $G_j$ ($j=1,2,\ldots,n$).

Step 3: Normalized the matrix $r=[A_{ij}]_{max}$ into $i=1,2,\ldots,m$ and $j=1,2,\ldots,n$ based on the type of each attribute by the following formula: For benefit attributes: $N_{ij}=A_{ij}=\left(\mu_{ij}, \eta_{ij}, v_{ij}\right)$ ($i=1,2,\ldots,m$ and $j=1,2,\ldots,n$)

For cost attributes: $N_{ij}=\left(\mu_{ij}, \eta_{ij}, v_{ij}\right)$ ($i=1,2,\ldots,m$ and $j=1,2,\ldots,n$)

Step 4: Multiply the normalized matrix $N_{ij}=\left(\mu_{ij}, \eta_{ij}, v_{ij}\right)$ ($i=1,2,\ldots,m$ and $j=1,2,\ldots,n$) and attributes weighting vector $w_j$ ($j=1,2,\ldots,n$), the picture fuzzy weighted normalized matrix $WN_{ij}(\mu_{ij}, \eta_{ij}, v_{ij})$ ($i=1,2,\ldots,m$ and $j=1,2,\ldots,n$) can be computed as:

$$WN_{ij} = \left(q \left[1 - \left(1 - \left(\mu_{ij}q \right)^{w_j} \eta_{ij}^{w_j} \right) \right], v_{ij}^{w_j}\right)$$

Step 5: Compute the values of bored approximation area and the bored approximation matrix $G=[g_{ij}]_{max}$ can be constructed as follows:

$$g_j = \left(\prod_{i=1}^{m} WN_{ij}\right)^{1/m} = \left(\prod_{i=1}^{m} \left(\mu_{ij}^{\prime}\right)^{1/m} \left[1 - \prod_{i=1}^{m} \left(1 - \left(\mu_{ij}q \right)^{w_j} \eta_{ij}^{w_j} \right) \right]\right)^{1/m}$$

Step 6: Compute the $D=[d_{ij}]_{max}$ between alternatives and the BAA by the following equation:

$$d_{ij} = \begin{cases} d(WN_{ij}, g_j) & \text{if } (WN_{ij} > g_j) \\ 0 & \text{if } (WN_{ij} = g_j) \\ -d(WN_{ij}, g_j) & \text{if } (WN_{ij} < g_j) \end{cases}$$

Where $d(WN_{ij}, g_j)$ is the distance from $WN_{ij}$ to $g_j$ which can be calculated by $q$th-rung orthopair picture fuzzy Normalized Euclidean distance ($q$th-ROPFNED).

Step 7: Sum the values of each alternative's result $S_i$, the order of all alternatives can be derived, the largest evaluation result $S_j$, the better choice. Hence, we obtain that the $q$th-rung orthopair picture fuzzy MABAC model which deals more complicated problems, which indicates that this method is more suitable to applied management activities, and enrich the research of management theory.

VI. NUMERICAL EXAMPLE

Here we give an actual decision making application to select best construction Project by using the $q$th-rung orthopair picture fuzzy MABAC model. Here let us consider the four companies, and so $q=4$. Assume four construction projects $A_i$ ($i=1, 2, 3, 4$), where $A_i$ are the construction projects to be selected and three attributes to evaluate these construction projects, where $G_1$ be the human factors, $G_2$ be the energy cost factors and $G_3$ be the building materials and environmental factors. Hence the four possible construction projects $A_i$ ($i=1, 2, 3, 4$) are to be evaluated with $q$th-ROPFNs with the three criteria. Suppose expert's weighting vector is $(0.45, 0.55)$ and attribute's weighting vector is $(0.32, 0.35, 0.33)$.

Step 1: Given the $4$th-rung orthopair picture fuzzy evaluation matrix $R=[A_{ij}^4]_{max}=(\mu_{ij}^4, \eta_{ij}^4, v_{ij}^4)$, ($i=1,2,\ldots,n$ and $j=1,2,\ldots,m$) as below:
Step 2: According to 4th-ROPFWA operator and experts weights, we can utilize overall $A_{ij}^k$ to $A_{ij}$ to obtain the matrix $r=[A_{ij}]_{mn}$ ($i=1,2,...,m$ and $j=1,2,...,n$) as below ($A_{11}$ for example):

$A_{11}=4\text{th}-\text{ROPFWA }[(0.8, 0.4, 0.6),(0.5,0.6,0.7)]$

$$q\prod_{j=1}^{n}(1-(\mu_{ij}^{w_j})^{q})\prod_{j=1}^{n}\eta_{ij}^{w_j}, \prod_{j=1}^{n}V_{ij}^{w_j} = \left(4\prod_{j=1}^{n}(1-0.8^{0.4})^{0.45} \prod_{j=1}^{n}(1-0.5^{0.45}) \prod_{j=1}^{n}(0.4^{0.45} \times 0.6^{0.55}) \prod_{j=1}^{n}(0.6^{0.45} \times 0.7^{0.55})\right)$$

$$= (0.6989, 0.4999, 0.6531)$$

$$\therefore R = \begin{bmatrix}
(0.6989,0.4999,0.6531) & (0.4,0.6,0.4781) & (0.71430.3973,0.4) \\
(0.4,0.3775,0.4801) & (0.5623,0.6531,0.7533) & (0.8173,0.2928,0.3775) \\
(0.5623,0.5442,0.4098) & (0.4,0.3,0.5) & (0.5990,0.3279,0.6929) \\
(0.5387,0.7533,0.6475) & (0.5623,0.5856,0.3512) & (0.5387,0.4801,0.4665)
\end{bmatrix}$$

Step 3: Normalize the matrix $r=[A_{ij}]_{mn}$, $i=1,2,...,m$ and $j=1,2,...,n$ based on the type of each attribute by the following formula, for $G_1$ is the cost attribute, thus $A_{12}, A_{22}, A_{32}$ need to be normalized, for that $N_{ij}=(A_{ij})=(\eta_A, \mu_A, \eta_A)$.

$\therefore N_{ij} = \begin{bmatrix}
(0.6989,0.4999,0.6531) & (0.4781,0.6,0.4) & (0.7143,0.3973,0.4) \\
(0.4,0.3775,0.4801) & (0.7533,0.6531,0.5623) & (0.8173,0.2928,0.3775) \\
(0.5623,0.5442,0.4098) & (0.5,0.3,0.4) & (0.5990,0.3279,0.6929) \\
(0.5387,0.7533,0.6475) & (0.3512,0.5856,0.5623) & (0.5387,0.4801,0.4665)
\end{bmatrix}$

Step 4: Multiply the normalized matrix $N_{ij}=(\mu_{ij}, \eta_{ij}, V_{ij})$ ($i=1,2,...,m$ and $j=1,2,...,n$) and attributes weighting vector $w_j$ ($j=1,2,...,n$), the picture fuzzy weighted normalized matrix $WN_{ij}$ can be computed as ($W_{11}$ for example):

$$WN_{ij} = \frac{q}{j} \prod_{j=1}^{n}(1-(\mu_{ij}^{w_j})^{q})^{\eta_{ij}^{w_j}}, V_{ij}^{w_j}$$

$$WN_{11} = \frac{1}{q} \sqrt[3]{1-(0.6989^{0.4})^{0.45}, 0.4999^{0.45}, 0.6531^{0.45}}$$

$$= \frac{1}{q}(0.0835, 0.8010, 0.8725)$$

$$= (0.5376, 0.8010, 0.8725)$$

Similar calculations are suitable for others. Hence the derived results of $WN_{ij}$ as below:
\[ WN_{ij} = \begin{bmatrix} (0.5376,0.8010,0.8725) & (0.3693,0.8363,0.7256) & (0.5547,0.7374,0.7391) \\ (0.9979,0.7322,0.7907) & (0.5972,0.8615,0.8175) & (0.6488,0.6668,0.7251) \\ (0.4269,0.8231,0.7517) & (0.3864,0.6561,0.7256) & (0.4590,0.6921,0.8817) \\ (0.4083,0.9133,0.8702) & (0.2698,0.8292,0.8175) & (0.4112,0.7849,0.7775) \end{bmatrix} \]

Step 5:
Compute the values of bored approximation area and the bored approximation matrix \( G = [g_{ij}]_{m \times n} \) can be constructed as follows:

\[
g_j = \left( \prod_{i=1}^{m} WN_{ij} \right)^{\frac{1}{m}} = \left[ \left( \prod_{i=1}^{m} \mu_{ij} \right)^{\frac{1}{m}}, \sqrt[4]{1 - \prod_{i=1}^{m} (1 - \eta_{ij})^{\frac{1}{m}}, \sqrt[4]{1 - \left( v_{ij}^{4} \right)^{\frac{1}{m}}} \right] \]

\[
g_1 = \left[ \sqrt[4]{0.5376 \times 0.9979 \times 0.4269 \times 0.4083}, \sqrt[4]{1 - \left[ (1 - 0.8010^4)^{\frac{1}{4}} \times (1 - 0.7322^4)^{\frac{1}{4}} \times (1 - 0.7907^4)^{\frac{1}{4}} \times (1 - 0.7517^4)^{\frac{1}{4}} \right]} \right] \]

\[
g_1 = (0.5530,0.8356,0.8314) \]

\[
g_2 = (0.3894,0.8144,0.7786) \]

\[
g_3 = (0.5105,0.7268,0.7965) \]

Step 6: Compute the distance \( D = [d_{ij}]_{m \times n} \) between alternatives and the BAA by equation in the algorithm: \( d_{11} \) for example

\[
d_{ij} = \begin{cases} d(WN_{ij}, g_j) & \text{if } S(WN_{ij}) > S(g_j) \\ 0 & \text{if } S(WN_{ij}) = S(g_j) \\ -d(WN_{ij}, g_j) & \text{if } S(WN_{ij}) < S(g_j) \end{cases} \]

\[ S(WN_{11}) = (1 + 0.5376^4 \times 0.8010^4 \times 0.8725^4) / 3 = 0.0308. \]

\[ S(g_1) = (1 + 0.5530^4 \times 0.8356^4 \times 0.8314^4) / 3 = 0.0427. \]

\[ \pi(WN_{11}) = \sqrt[4]{0.5376^4 + 0.8010^4 + 0.8725^4 - (0.5376^4 \times 0.8010^4 \times 0.8725^4)} \]

\[ \pi(WN_{11}) = 1.0134. \]

\[ \pi(g_1) = \sqrt[4]{0.5530^4 + 0.8356^4 + 0.8314^4 - (0.5530^4 \times 0.8356^4 \times 0.8314^4)} \]

\[ \pi(g_1) = 1.0091. \]
\[ d_{11} = d_{\text{ROPFNE}}^e (WN_{11}, g_1) \]

\[ d_{11} = \sqrt{\frac{1}{2(3)(3)} \left[ 0.5376^4 - 0.5530^4 |^2 + | 0.8010^4 - 0.8356^4 |^2 + | 0.8725^4 - 0.8314^4 |^2 + | 1.0134^4 - 1.0091^4 |^2 \right] } \]

\[ d_{11} = \sqrt{0.00095} = -0.0308, \quad \text{Since } S(WN_{11}) < S(g_1) \]

Table: The distances \( d_i \) between alternatives and BAA

<table>
<thead>
<tr>
<th>A</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.0018</td>
<td>0.0265</td>
<td>0.0295</td>
</tr>
<tr>
<td>A2</td>
<td>0.2502</td>
<td>-0.0748</td>
<td>0.0497</td>
</tr>
<tr>
<td>A3</td>
<td>0.0678</td>
<td>0.1030</td>
<td>-0.0581</td>
</tr>
<tr>
<td>A4</td>
<td>-0.0813</td>
<td>-0.0507</td>
<td>-0.0581</td>
</tr>
</tbody>
</table>

Step 7: Sum the values of each alternative’s \( d_i \) : (S1 for example)

\[ S_1 = d_{11} + d_{12} + d_{13} = 0.0308 + 0.2502 + 0.0295 = 0.3105 \]

\[ S_2 = 0.22511 \]

\[ S_3 = 0.1127 \]

\[ S_4 = -0.1396. \]

The order of all alternatives can be derived. Clearly, the biggest one is the better choice.

Hence the order is \( S_2 > S_3 > S_1 > S_4 \)

(i.e.,) \( A_2 > A_3 > A_1 > A_4 \)

Therefore \( A_2 \) is the ideal solution.

Here we conclude that, among four construction projects with three attributes such as human factors, energy cost factor and building environmental factors, \( A_2 \) is considered as the best construction project by using 4th-rung orthopair picture fuzzy set with MABAC model.

REFERENCES