THE ENERGY OF MINIMUM DOMINATION IN GRAPHS

A.Nagarani[1], S.Vimala[2]

Research Scholar, Department of Mathematics, Mother Teresa Women's University[1]

Assistant Professor, Department of Mathematics, Mother Teresa Women's University[2]

nagarani25kodai@gmail.com[1] tvimss@gmail.com[2]

Abstract **- The energy calculated in domination graphs is defined in [11].In this paper, we derived minimum domination cover graphs. For the minimum covering graph found energy, E_C (G) of a graph which depends on its particular minimum cover C.**

AMS Classification **- 05C50, 05C69**

Keyword **- Minimum dominating set, minimum dominating matrix, minimum dominating eigen values, minimum dominating energy of a graph**.

I. INTRODUCTION

The mathematical study of Domination Theory in graphs started around 1960. The concept of energy of a graph was introduced by I.Gutman [3,6] in the year 1978. The roots go back to 1862 when C.F. De Jaenisch studied the problem of determining the minimum number of queens necessary to cover an n × n chess board in such way that every square is attacked by one of the queens. Energy of graphs defined for regular [4], non-regular [9], circulate [12] and random graphs [2], signed graphs in [6] and for weighted graph by I. Gutman and Shao in 2011. In [1] R. Balakrishnan defined the energy of π electrons of the molecule is approximately the energy of its molecular graph. The basic properties including various upper and lower bounds for energy of a graph have been established in [7,10], and it has found remarkable chemical application in the molecular orbital theory of conjugated molecules[5,6]. In this paper we derived minimum dominating energy of a graph $E_D(G)$ in Cycle graph, Star graph, Wheel graph, Regular graph and also calculate Upper and Lower bounds of energy

II. PRELIMINARIES

2.1 Labeling

A labeling of a graph is an assignment of values to the vertices and edges of a graph.

2.2 Vertex Labeling

Given a graph G, an injective function f :V(G)→ N has been called a vertex labeling of G.

2.3 Edge Labeling

An edge labeling of a graph is a bijection from E(G) to the set {1, 2. … ,|E(G| }.

2.4 Energy

Energy of a simple graph G = (V, E) with adjacency matrix A is defined as the sum of absolute values of eigen values of A denoted by E (G). ie, E(G) $=\sum_{i=1}^{n}|\lambda_i|$ where λ_i is an eigen values of A, i =1, 2, . . . , n.

2.5 Dominating set: [11,8]

A subset S of V(G) is said to be dominating set if for every vertex v in V(G)-S, there is a vertex u in S such that u is adjacent to v. That is a vertex v of G is in S or is adjacent to some vertex of S.

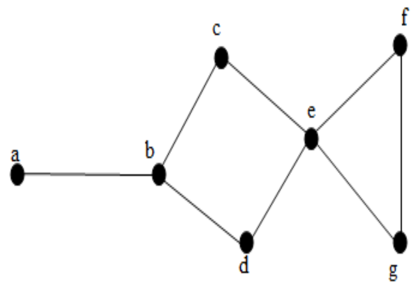

Fig.1

For instance the vertex set {b, g} is a dominating set in this Graph of Fig.1 The set {a, b,c ,d , f} is a dominating set of the graph G. For a graph G, G-{v} denote the graph obtain by removing vertex v and all edges incident to v.

2.6 Minimum Dominating Set[11].

A dominating set with least number of vertices is called minimum dominating set. It is denoted as γ set of the graph G.

2.7 Domination Number.[8]

The number of vertices in a minimum dominating set is called domination number of the graph G. It is denoted by γ(G).

2.8 The Minimum Dominating Energy[11]

Let G be a simple graph of order n with vertex set V ={$v_1, v_2, \dots, v_n$} and edge set E. A subset D of V is called a dominating of G if every vertex of V-D is adjacent to some vertex in D. Any dominating set with minimum cardinality is called a minimum dominating set. Let D be a minimum dominating set of a graph G. The minimum matrix of G is the n× n matrix defined by $A_D(G) = (a_{ij})$, where

$$a_{ij} = \begin{cases} 1 & \text{if } v_i v_j \in E, \\ 1 & \text{if } i = j \text{ and } v_i \in D, \\ 0 & \text{otherwise.} \end{cases}$$

The characteristic polynomial of $A_D(G)$ is denoted by $f_n(G,\lambda) = \det(\lambda I - A_D(G))$. The minimum dominating eigen values of the graph G are the eigen values of $A_D(G)$. Since $A_D(G)$ is real and symmetric, its eigen values are real numbers and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \dots \geq, \lambda_n$. The minimum dominating energy of G is defined as

$$E_D(G) = \sum_{i=1}^{n} |\lambda_i|$$

The trace of $A_D(G)$ = Dominating Number = k

2.9 Wheel graph

A Wheel graph W_n is a graph formed by connecting a single vertex to all vertices of a cycle, and otherwise disjoint.

2.10 Star graph

The Star S_n of order n, sometimes simply know as an n –star , is a tree on n nodes with one node having vertex degree n-1 and the other n-1 havinh vertex degree 1.

2.11 Regular Graph

A regular graph is a graph where each vertex has the same number of neighbors

2.12 Wheel Grah

A Wheel graph is a graph formed by connecting a single vertex to all vertices of a cycle. A wheel graph with n vertices can also be defined as the 1-skeleton of an (n-1)-gonal pyramid.

III. MAIN RESULT

Theorem 3.1.

If complete graph K_n satisfies energy of every singleton minimum dominating is constant .

Proof

To prove that the energy of complete graph with minimum dominating sets are constant.

Let $v_1,v_2,v_3,v_4,v_{5,\ldots,}v_n$ are the vertices of the complete graph K_n

It's clear that every singleton set in a complete graph is a minimum dominating set, since every vertex of K_n is adjacent to all other vertices of K_n

We compute the adjacency matrix

$A_D(G) = (a_{ij})$, where

$$a_{ij} = \begin{cases} 1 & \text{if } v_i v_j \in E, \\ 1 & \text{if } i = j \text{ and } v_i \in D, \\ 0 & \text{otherwise.} \end{cases}$$

its eigen values are real numbers and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \ldots . \geq , \lambda_n$. The minimum dominating energy of G is defined as

$$E_D(G) = \sum_{i=1}^{n} |\lambda_i|$$

The trace of $A_D(G)$ = Dominating Number = k

The minimum dominating energy of complete graph is constant

Example 1:

Let K_5 be a complete graph with vertex v_1,v_2,v_3,v_4, v_5.

The minimum dominating sets are $\{v_1\},\{v_2\},\{v_3\},\{v_4\},\{v_5\}$

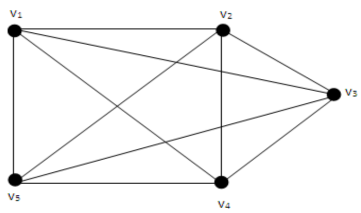

Fig 1

the minimum dominating adjacency matrix are

$$A_{D1}(G) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \qquad A_{D2}(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$A_{D3}(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \qquad A_{D4}(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

The minimum dominating eigen values of the minimum dominating adjacency matrix are $A_{D1}(G)$, $A_{D2}(G)$ $A_{D3}(G)$ $A_{D1}(G)$ = -1.0000, -1.0000, -1.0000, -0.2361, 4.2361

The minimum dominating energy are = 7.4722

Theorem 3.2.

If circle graph C_n satisfies energy of minimum dominating set is same value depends upon the minimum dominating adjacent matrix

Proof

To prove that a circle graph has same energy in minimum dominating set.

Let $v_1,v_2,v_3,v_4,v_{5,\ldots},v_n$ are the vertices of the circle graph C_n

Let we take a minimum dominating set

We compute the adjacency matrix

$A_D(G) = (a_{ij})$, where

$$a_{ij} = \begin{cases} 1 & if\ v_i v_j \in E, \\ 1 & if\ i = j\ and\ v_i \in D, \\ 0 & otherwise. \end{cases}$$

its eigen values are real numbers and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \ldots . \geq , \lambda_n$. The minimum dominating energy of G is defined as

$$E_D(G) = \sum_{i=1}^{n} |\lambda_i|$$

The trace of $A_D(G)$ = Dominating Number = k

The minimum dominating energy of complete graph is constant

Example 2:

Let C_5 be a circle graph with vertex v_1,v_2,v_3,v_4,v_5.

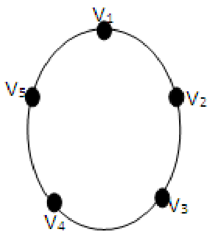

Fig 2

In this graph(fig 2) has the minimum dominating set are $\{v_1v_3\}$,$\{v_1v_4\}$is minimal . Then (C_5) =2.

The minimum dominating adjacency matrix are

$$A_{D1}(G) = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad A_{D2}(G) = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The minimum dominating eigen values of the minimum dominating adjacency matrix are $A_{D1}(G)$, $A_{D2}(G)$ = -1.4142,-1.1701,0.688

The minimum dominating energy of circle graph C_5 = 7.1686

Theorem 3.3.

Show that the energy of Wheel graphs and Star graphs depends on the centre vertex of G.

Proof:

Since the centre vertex of wheel graph and star graph are minimum dominating set of G

Therefore minimum dominating energy of wheel graph and star graph depends on the centre vertex of G.

Example 3:

Let S_9 be a Star graph with vertex $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9$

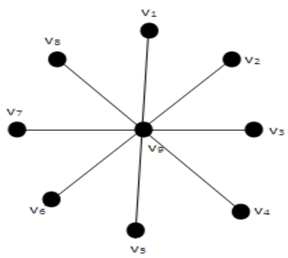

Fig 3

In this graph(fig 3) has the minimum dominating set is $\{v_9\}$is minimal . Then (S_9) =1.

The minimum dominating adjacency matrix are

$$A_{D1}(G) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The minimum dominating eigen values of the minimum dominating adjacency matrix is

$A_{D1}(G)$ = -2.3723,0, 0, 0, 0, 0, 0, 0, 3.3723

The minimum dominating energy of Star graph S_9 = 5.7446

The energy depends upon the centre vertex of star graph.

Example 4:

Let W_9 be a Wheel graph with vertex $v_1,v_2,v_3,v_4,v_5, v_6,v_7,v_8, v_9$

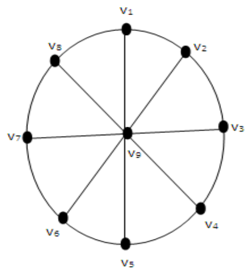

Fig 4

In this graph(fig 4) has the minimum dominating set is $\{v_9\}$is minimal . Then $W_9 =\{1\}$.

The minimum dominating adjacency matrix are

$$A_{D1}(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The minimum dominating eigen values of the minimum dominating adjacency matrix is

$A_{D1}(G)$ = -2.0000,-1.4142, -1.4142, -1.3723, -0.0000, 0.0000, 1.4142,1.4142,4.3723

The minimum dominating energy of Wheel graph W_9 = 13.4014

The energy depends upon the centre vertex of Wheel graph.

IV. Conclusion:

In this paper the minimum domination energy defined of few graphs. The authore extend and to find upper and lower bound of minimum dominating graphs.

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