Queueing systems with C - servers under differentiated type 1 and type 2 vacations

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***Abstract*—** **Multiple vacations queueing systems played an enormous role in the enhancement of several excellent research works in the field of queueing theory and its related applications. But most of the queueing models with multiple vacation policies were emerged by assuming that the consecutive vacations offered for the servers in the system will follow the same distribution with the same mean. But for a few years back a new idea named differentiated vacation was originated to distinguish the successive vacations offered in a single server queueing system. In this paper we introduced the concept of differentiated vacations for the queueing system with multiple servers. In the proposed queueing model we allowed the servers to take two types of vacations, namely type 1 and type 2 vacations. We formulated the equations for calculating the steady state probabilities and the average waiting time generated in the system. Further we investigated the nature of the average waiting time with different durations of vacations and also identified the relation between the durations of vacations and the number of servers in the system.**

***Keywords*—** **multiple server queueing systems, vacation queueing systems, multiple vacations, differentiated vacations, average waiting time.**

I. INTRODUCTION

Queueing systems with server vacations opened a wide platform for the new explorations in queueing theory. Server vacations queueing systems possess tremendous role in our daily life situations. Now a days the main aim of service providers in all streams is to offer better service for the customers with in a limited time due to the tough competitions occur in the field of service providing. The concept of vacations in to the traditional model of queueing systems was first introduced by the researchers Levy and Yenchiali in 1975[8]. The queueing systems with server vacation has the feature that the server become unavailable for a certain period of time from the respective service area and the time that the server become unavailable is termed as a vacation in queueing systems. Mainly there are some reasons for the servers to take a vacation like power saving mode, machine breakdowns, network problems, unavailability of power, physical problems for servers, coffee breaks etc. Queueing systems with server vacations affect the stability of the system noticeably if we didn't handle the vacation period correctly.

The attention of many researchers has attracted in to vacation queueing systems and it leads to originate several excellent surveys on the respective concept, in which the survey given by B. T. Doshi [3,4] was discussed deeply about vacations in a single server queueing systems and at the end of his work he concluded that many queueing problems and their solutions can be simplified if viewed as vacation type problems. The researchers like Takagi [13] and Zhang[15] have affectionated into the vacation policies in queueing systems and extend this notion a while.

Single vacation scheme and multiple vacations scheme are the two major divisions of vacation queueing systems. We concentrated on multiple vacations queueing systems throughout this paper. There is an exceptional case that at the time of vacation due to some special concern the server can active with a different performance rate. This occasion was expanded by many researchers in the name of working vacations and gave attractive inventions in to the theory of queueing systems. Working vacation scheme was introduced by Servi and Finn in 2002 [10] and the model introduced by them was generalized into a M/G/1 queue with general working vacation by Wu and Takagi [17]. Baba[1] discussed about working vacation scheme in G1/M/1 queue by matrix analytic method. Banik et al [2] studied about the GI/M/1/N queue with multiple vacations. Liu et al [9] entrenched a stochastic decomposition result in single server queueing systems with working vacation.

Most of the scientists like Tian et al [16], Zhang and Hau [18] who worked with multiple vacation schemes assumed that the consecutive vacations in their proposed model follow the same distribution. But Ibe and Isijola [7] studied about M/M/1 multiple vacation queueing systems where vacations follow different durations. Those type of vacations are termed as differentiated vacations. The notation of differentiated vacation are also introduced in [6,12] in the framework of gated vacation while the multiple adaptive vacations are presented in [14] in a discrete time context. Gupta et al [5] have studied about M/D/1 multiple vacation queueing systems with deterministic service time using the concept of differentiated vacation. In 2018 Vijayashree and Janani [11] analysed about the transient state solution of an M/M/1 queueing system subjected to differentiated vacations.

In the proposed queueing model we extend differentiated vacation scheme in to a multiple server queueing system. We allow the servers to go for two types of vacations in the system; in which the first type of vacations can be taken after completing a nonzero busy period of each servers of the system. Whereas the second type can be taken after busy period of zero duration. But for the first and second type of vacations there are separate timing and fixed durations for all servers in the system. The main advantage of this model is that it helps to avoid the idle characters of the servers and hence the performance level of them gets increased. As a result more customers will arrive for the service because of the less waiting time in the system. For example these models can be introduced in the administrative sessions in multinational companies, transaction section of banks, ticket counters of railway stations, theaters etc to avoid the working pressure of the employees and to improve the efficiency of them.

The rest of the paper is organized as follows. Section 2 is the detailed interpretation of the proposed model and the derivations of the important measures of queueing system are included. In section 3 we discussed some numerical examples with graphical interpretations and the obtained results are concluded in section 4.

II. System modelling and analysis

We consider a $M/M/C$ queueing system with multiple vacations where customers arrive according to Poisson rate λ and the time to serve a customer is exponentially distributed with mean $\frac{1}{\mu}$, where $\mu > \lambda$.We assumed that there are two type of vacations are possible for each server, namely type 1 and type 2 vacations. The former is a vacation taken by the servers after completing a nonzero busy period, whereas the later can be taken after a zero busy peroid. In other words the servers can go for a vacation of type 1 only after serving at least one customer in the system and type 2 vacation can be taken by the servers if no customers are waiting in any of the C queues for service when they returned from the vacation. Note that servers can repeat the type 2 vacations till at least one customer arrives to in any of the C queues. As mentioned earlier in the proposed model distribution followed by the durations of type 1 vacations is entirely different from the distribution followed by the durations of type 2 vacations and the durations of type 1 vacations are independent of busy period of the servers. We also assumed that the durations of type 1 and type 2 vacations follow exponential distributions with means $\frac{1}{\gamma_1}$ and $\frac{1}{\gamma_2}$ respectively.

Let us denote the state of the system by (n, k), where n is the number of customers in the system. By assuming some fixed values for k so as to provide information regarding the availability of servers and the nature of vacations, we observe that if $k = 0$, all the servers are active, $k = 1$, all the servers are in type 1 vacation and $k = 2$, all the servers are in type 2 vacation. Hence the system can be modelled by a continuous time Markov chain with the state transition diagram drawn as in figure 1.

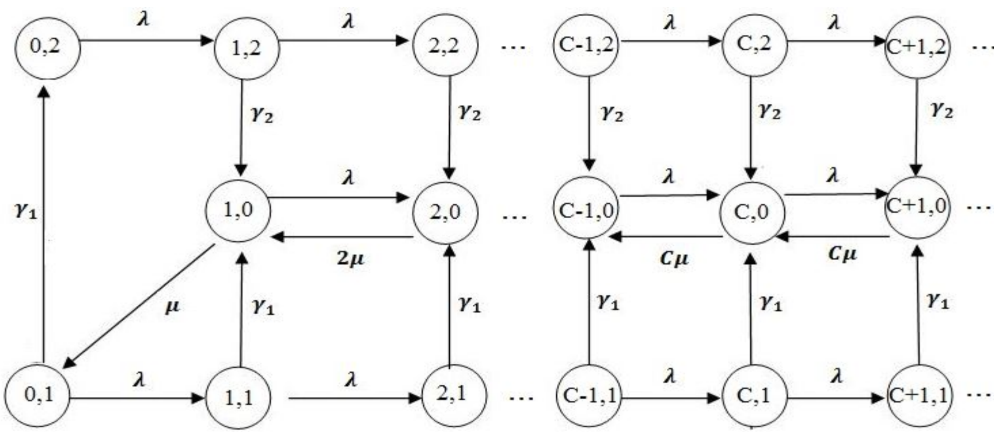

Fig 1. State Transition Diagram

Theorem 1: The steady state probabilities $P_{n,k}$ for the different values of k are given by

(i)

$$P_{n,0} = \begin{cases} \left(\dfrac{\rho^{n-1}}{n!} + \displaystyle\sum_{i=0}^{n-2} \dfrac{\rho^{n-i-1}(i+1)!}{n!}\left[\alpha_1\beta_1^{i+1} + \alpha_2\beta_2^{i+1}\right]\right) P_{1,0} & n \le C \\ \left(\left(\dfrac{\rho}{C}\right)^{n-C} A(C,0) + \displaystyle\sum_{i=1}^{n-C} \left(\dfrac{\rho}{C}\right)^{i}\left[\alpha_1\beta_1^{n-i} + \alpha_2\beta_2^{n-i}\right]\right) P_{1,0} & n > C \end{cases}$$

(ii) $P_{n,1} = \alpha_1 {\beta_1}^n P_{1,0}$

(iii) $P_{n,2} = \alpha_2 {\beta_2}^n P_{1,0}$

where C is the number of servers in the system, $\rho = \frac{\lambda}{\mu}$ is the offered load, $\alpha_1 = \frac{\mu}{(\lambda+\gamma_1)}$, $\alpha_2 = \frac{\gamma_1\mu}{\lambda(\lambda+\gamma_1)}$,

$\beta_1 = \frac{\lambda}{(\lambda+\gamma_1)} < 1$, $\beta_2 = \frac{\lambda}{(\lambda+\gamma_2)} < 1$, $A(C,0) = \frac{\rho^{C-1}}{C!} + \sum_{i=0}^{C-2} \frac{\rho^{C-i-1}(i+1)!}{C!}\left[\alpha_1\beta_1^{i+1} + \alpha_2\beta_2^{i+1}\right]$ and

$$P_{1,0} = \left(\sum_{n=1}^{C}\left(\frac{\rho^{n-1}}{n!} + \sum_{i=0}^{n-2}\frac{\rho^{n-i-1}(i+1)!}{n!}\left[\alpha_1\beta_1^{i+1} + \alpha_2\beta_2^{i+1}\right]\right) + \frac{\rho}{C-\rho}\left(A(C,0) + \frac{\alpha_1\beta_1^C}{1-\beta_1} + \frac{\alpha_2\beta_2^C}{1-\beta_2}\right) + \frac{\alpha_1}{1-\beta_1} + \frac{\alpha_2}{1-\beta_2}\right)^{-1}$$

Proof:

By considering global balance in figure 1 we obtain

$$\mu P_{1,0} = (\lambda + \gamma_1) P_{0,1} \quad (1)$$

$$\gamma_1 P_{0,1} = \lambda P_{0,2} \quad (2)$$

Thus

$$P_{0,1} = \frac{\mu}{(\lambda+\gamma_1)} P_{1,0} = \alpha_1 P_{1,0} \quad (3)$$

$$P_{0,2} = \frac{\gamma_1}{\lambda} P_{0,1} = \alpha_2 P_{1,0} \quad (4)$$

where $\alpha_1 = \frac{\mu}{(\lambda+\gamma_1)}$, $\alpha_2 = \frac{\gamma_1 \mu}{\lambda(\lambda+\gamma_1)}$. In a similar way we obtain for $n = 0,1,2 \ldots$

$$\lambda P_{n,1} = (\lambda + \gamma_1) P_{n+1,1}, \quad (5)$$

$$\lambda P_{n,2} = (\lambda + \gamma_2) P_{n+1,2}, \quad (6)$$

which gives for $n = 0,1,2 \ldots$,

$$P_{n+1,1} - \frac{\lambda}{\lambda+\gamma_1} P_{n,1}, \quad (7)$$

$$P_{n+1,2} = \frac{\lambda}{\lambda+\gamma_2} P_{n,2}, \quad (8)$$

By solving equations (7) and (8) recursively we get

$$P_{n,1} = \alpha_1 {\beta_1}^n P_{1,0}, \, , \quad (9)$$

and

$$P_{n,2} = \alpha_2 {\beta_2}^n P_{1,0}, \quad (10)$$

where $\beta_1 = \frac{\lambda}{(\lambda+\gamma_1)} < 1$, $\beta_2 = \frac{\lambda}{(\lambda+\gamma_2)} < 1$ and $n = 0,1,2 \ldots$

To analyse the steady state probabilities $P_{n,0}$ for the state $(n, 0)$ we have to consider two cases regarding the number of customers.

Case 1: $(n \leq C)$

In this case the number of customers cannot exceed the number of servers. Hence the local balance equations give,

$$\lambda P_{n-1,1} + \lambda P_{n-1,2} + \lambda P_{n-1,0} - n\mu P_{n,0} \, , \quad (11)$$

By using equations (9) and (10) and by letting $\rho = \frac{\lambda}{\mu}$, we can write the above equation as,

$$P_{n,0} = \frac{\rho}{n}\left(P_{n-1,0} + \alpha_1 {\beta_1}^{n-1} P_{1,0} + \alpha_2 {\beta_2}^{n-1} P_{1,0}\right), \quad (12)$$

Solving the equation (12) recursively we obtain,

$$P_{2,0} = \left(\frac{\rho}{2} + \alpha_1\beta_1\frac{\rho}{2} + \alpha_2\beta_2\frac{\rho}{2}\right)P_{1,0}$$

which in turn gives,

$$P_{3,0} = \left(\frac{\rho^2}{3!} + \alpha_1\beta_1\left[\frac{\rho^2}{3!} + \alpha_1\frac{\rho}{3}\right] + \alpha_2\beta_2\left[\frac{\rho^2}{3!} + \alpha_2\frac{\rho}{3}\right]\right)P_{1,0}$$

Hence in general we can write

$$P_{n,0} = \left(\frac{\rho^{n-1}}{n!} + \sum_{i=0}^{n-2}\frac{\rho^{n-(i+1)}(i+1)!}{n!}\left[\alpha_1\beta_1^{i+1} + \alpha_2\beta_2^{i+1}\right]\right)P_{1,0} \tag{13}$$

Case 2: $(n > C)$

When the number of customers exceeds the number of servers local balance equations becomes,

$$\lambda P_{n-1,0} + \lambda P_{n-1,1} + \lambda P_{n-1,2} = C\mu P_{n,0} \tag{14}$$

By using equations (9) and (10), we can write the above equation as,

$$P_{n,0} = \frac{\rho}{C}\left(P_{n-1,0} + \alpha_1{\beta_1}^{n-1}P_{1,0} + \alpha_2{\beta_2}^{n-1}P_{1,0}\right), \tag{15}$$

By solving equation (15) recursively we obtain,

$$P_{C+1,0} = \frac{\rho}{C}\left(P_{C,0} + \alpha_1{\beta_1}^{C}P_{1,0} + \alpha_2{\beta_2}^{C}P_{1,0}\right),$$

where

$$P_{C,0} = \left(\frac{\rho^{C-1}}{C!} + \sum_{i=0}^{C-2}\frac{\rho^{C-(i-1)}(i+1)!}{C!}\left[\alpha_1\beta_1^{i+1} + \alpha_2\beta_2^{i+1}\right]\right)P_{1,0} = A(C,0)P_{1,0} \quad \text{(say)} \tag{16}$$

By following the same procedure for calculating $P_{C+2,0}$ and $P_{C+3,0}$, we can generalize that

$$P_{n,0} = \left(\left(\frac{\rho}{C}\right)^{n-C}A(C,0) + \sum_{i=1}^{n-C}\left(\frac{\rho}{C}\right)^{i}\left[\alpha_1\beta_1^{n-i} + \alpha_2\beta_2^{n-i}\right]\right)P_{1,0}, \tag{17}$$

where $A(C,0) = \frac{\rho^{C-1}}{C!} + \sum_{i=0}^{C-2}\frac{\rho^{C-i-1}(i+1)!}{C!}\left[\alpha_1\beta_1^{i+1} + \alpha_2\beta_2^{i+1}\right].$ (18)

Now it remains to find the value of $P_{1,0}$. By applying the normality conditions,

$$\sum_{n=1}^{\infty}P_{n,0} + \sum_{n=0}^{\infty}P_{n,1} + \sum_{n=0}^{\infty}P_{n,2} = 1 \tag{19}$$

That is,

$$\sum_{n=1}^{C}P_{n,0} + \sum_{n=C+1}^{\infty}P_{n,0} + \sum_{n=0}^{\infty}P_{n,1} + \sum_{n=0}^{\infty}P_{n,2} = 1 \tag{20}$$

By using the equations (13), (17), (9) and (10) for the respective probabilities in (20) and by taking the respective summations with some algebraic calculations we obtain,

$$\left\{\sum_{n=1}^{C}\left(\frac{\rho^{n-1}}{n!} + \sum_{i=0}^{n-2}\frac{\rho^{n-i-1}(i+1)!}{n!}\left[\alpha_1\beta_1^{i+1} + \alpha_2\beta_2^{i+1}\right]\right) + \frac{\rho}{C-\rho}\left(A(C,0) + \frac{\alpha_1\beta_1^C}{1-\beta_1} + \frac{\alpha_2\beta_2^C}{1-\beta_2}\right) + \frac{\alpha_1}{1-\beta_1} + \frac{\alpha_2}{1-\beta_2}\right\}P_{1,0} = 1$$

Hence,

$$P_{1,0} = \left(\sum_{n=1}^{C}\left(\frac{\rho^{n-1}}{n!} + \sum_{i=0}^{n-2}\frac{\rho^{n-i-1}(i+1)!}{n!}\left[\alpha_1\beta_1^{i+1} + \alpha_2\beta_2^{i+1}\right]\right) + \frac{\rho}{C-\rho}\left(A(C,0) + \frac{\alpha_1\beta_1^C}{1-\beta_1} + \frac{\alpha_2\beta_2^C}{1-\beta_2}\right) + \frac{\alpha_1}{1-\beta_1} + \frac{\alpha_2}{1-\beta_2}\right)^{-1}$$

This completes the proof.

Theorem 2: The average queue length of the queueing system is given by,

$$E(m) = \left\{\left(\frac{C\rho}{(C-\rho)^2}\right)A(C,0) + \frac{\alpha_1\beta_1{}^{C+1}}{(1-\beta_1)^2} + \frac{\alpha_2\beta_2{}^{C+1}}{(1-\beta_2)^2} + \frac{\alpha_1\rho}{(C-\rho)(1-\beta_1)}\left\{\frac{C\beta_1^C-(C-1)\beta_1^{C+1}}{1-\beta_1} + \frac{C\beta_1^C(1-C-\rho)}{C-\rho}\right\} + \frac{\alpha_2\rho}{(C-\rho)(1-\beta_2)}\left\{\frac{C\beta_2^C-(C-1)\beta_2^{C+1}}{1-\beta_2} + \frac{C\beta_2^C(1-C-\rho)}{C-\rho}\right\}\right\}P_{1,0}$$

.

where C is the number of servers in the system, $\rho = \frac{\lambda}{\mu}$ is the offered load, $\alpha_1 = \frac{\mu}{(\lambda+\gamma_1)}$, $\alpha_2 = \frac{\gamma_1\mu}{\lambda(\lambda+\gamma_1)}$,

$\beta_1 = \frac{\lambda}{(\lambda+\gamma_1)}<1$, $\beta_2 = \frac{\lambda}{(\lambda+\gamma_2)}<1$,

$A(C,0) = \frac{\rho^{C-1}}{C!} + \sum_{i=0}^{C-2}\frac{\rho^{C-i-1}(i+1)!}{C!}\left[\alpha_1\beta_1^{i+1} + \alpha_2\beta_2^{i+1}\right]$ and

$$P_{1,0} = \left(\sum_{n=1}^{C}\left(\frac{\rho^{n-1}}{n!} + \sum_{i=0}^{n-2}\frac{\rho^{n-i-1}(i+1)!}{n!}\left[\alpha_1\beta_1^{i+1} + \alpha_2\beta_2^{i+1}\right]\right) + \frac{\rho}{C-\rho}\left(A(C,0) + \frac{\alpha_1\beta_1^C}{1-\beta_1} + \frac{\alpha_2\beta_2^C}{1-\beta_2}\right) + \frac{\alpha_1}{1-\beta_1} + \frac{\alpha_2}{1-\beta_2}\right)^{-1}$$

Proof:

The average queue length of a queueing system with C service channels is given by

$$E(m) = \sum_{n=C}^{\infty}(n-C)P_n$$

Hence in this case we have

$$E(m) = \sum_{n=C}^{\infty}(n-C)P_{n,0} + \sum_{n=C}^{\infty}(n-C)P_{n,1} + \sum_{n=C}^{\infty}(n-C)P_{n,2} \quad (21)$$

Consider the summations in the above equation separately and by evaluating them with some algebraic calculations we obtain the following equations.

$$\sum_{n=C}^{\infty}(n-C)P_{n,1} = \left(\frac{\alpha_1\beta_1^{C+1}}{(1-\beta_1)^2}\right)P_{1,0} \quad (22)$$

$$\sum_{n=C}^{\infty}(n-C)P_{n,2} = \left(\frac{\alpha_2\beta_2^{C+1}}{(1-\beta_2)^2}\right)P_{1,0} \quad (23)$$

$$\sum_{n=C}^{\infty}(n-C)P_{n,0} = \left\{\left(\frac{C\rho}{(C-\rho)^2}\right)A(C,0) + \frac{\alpha_1\rho}{(C-\rho)(1-\beta_1)}\left\{\frac{C\beta_1^C-(C-1)\beta_1^{C+1}}{1-\beta_1} + \frac{C\beta_1^C(1-C-\rho)}{C-\rho}\right\} + \frac{\alpha_2\rho}{(C-\rho)(1-\beta_2)}\left\{\frac{C\beta_2^C-(C-1)\beta_2^{C+1}}{1-\beta_2} + \frac{C\beta_2^C(1-C-\rho)}{C-\rho}\right\}\right\}P_{1,0} \quad (24)$$

By using the equations (22), (23) and (24) in (21) we get the required formula for $E(m)$, which completes the proof.

III. NUMERICAL ANALYSIS

In the proposed model of queueing system we investigated about the impacts of variations made in the duration of vacations on the average waiting time. Throughout this discussion we fixed the parameter μ as 0.25 and considered the number of servers (C) as two and four respectively. In first case let us assume that the mean duration $1/\gamma_1$ of type 1 vacation is much higher than the mean duration $1/\gamma_2$ of type 2 vacation, where type1 vacations have offered for the servers after completing a non zero busy period of each of them and type 2 vacation can be taken if there are no customers are waiting in any of the C queues for service. Now by fixing the value of γ_2 as 1 and varying the value of γ_1 as 0.05 and 0.07 respectively we have evaluated the average waiting time $(E(v))$ in the system for each pair of (γ_1, γ_2) for some selected values of ρ and for each value of C. We also calculated $E(v)$ by changing the value of γ_2 as 2 and keeping the values of γ_1 as 0.05 and 0.07. The obtained values of $E(v)$ are tabulated in table 1. We used Mathematica 5.2 software to evaluate the values of $E(v)$ in all the cases.

Table I: MEAN TIME IN THE SYSTEM FOR VARIOUS ρ

γ_2	ρ	$E(v)$			
		C=4		C=2	
		$\gamma_1 = 0.07$	$\gamma_1 = 0.05$	$\gamma_1 = 0.07$	$\gamma_1 = 0.05$
1	0.2	4.2216	4.7755	5.5004	7.4859
	0.5	5.7335	8.1193	9.3662	13.644
	0.8	7.2847	10.7185	12.2715	17.3032
	0.9	7.7194	11.3754	13.1159	18.2787
2	0.2	4.2239	4.7815	5.5113	7.5095
	0.5	5.7496	8.1439	9.4050	13.6934
	0.8	7.3090	10.7489	12.3166	17.3516
	0.9	7.7451	11.4063	13.1609	18.3255

In order to observe the behaviour of the average waiting time in the system for each value of C with the respective fixed parameters we plotted $E(v)$ by varying ρ in each case. Figure 2 depicts the variation of average waiting time $(E(v))$ in the system against the offered load (ρ) for $\gamma_2 - 1$ and γ_1 = 0.05,0.07. The corresponding figure reveals that $E(v)$ decreases when γ_1 increases from 0.05 to 0.07. So it can be concluded that for each selected values of C, $E(v)$ increases when the value of $1/\gamma_1$ increases. The graphical interpretation also shows that in a 2 servers queueing system $E(v)$ increases very quickly when compared to the growth of $E(v)$ in the 4 servers queueing system. So the suggested system will work with an appropriate rate of average waiting time if we appoint 4 servers to perform the respective duties in the system. Also we have observed that a minor change in the value of γ_1 affect the average waiting time in the system recognizably. Figure 3 describes the variations of $E(v)$ against ρ for the fixed values of the parameters, γ_2 as 2 and $\gamma_1$takes 0.05 and 0.07. It is evident that the observations regarding the values of $E(v)$ is analogous to that of previous case. While observing the values of $E(v)$ given in table 1 we can note that for each value of C and γ_1, when the value of γ_2 changes from 1 to 2 there occur a slight increment in the corresponding value of $E(v)$. By fixing the parameters γ_1 and ρ we can procure some important observations related to the range of the mean duration of type 2 vacations.

Table 2: MEAN TIME IN THE SYSTEM FOR VARIOUS γ_2 AND $\rho = 0.05$

γ_2	$E(v)$			
	$\gamma_1 = 0.05$		$\gamma_1 = 0.07$	
	C=2	**C=4**	**C=2**	**C=4**
0.08	13.7152	8.0114	10.2186	5.9810
0.09	13.5773	7.955	9.9377	5.8527
0.1	13.4917	7.9251	9.7421	5.7731
0.4	13.5169	8.0496	9.2820	5.6896
0.5	13.5553	8.0720	9.3042	5.7033
0.9	13.6334	8.1139	9.3583	5.7300
1	13.644	8.1193	9.3662	5.7335
2	13.6934	8.1439	9.4050	5.7496
3	13.7106	8.1522	9.419	5.7551

Consider the values of $E(v)$ given in table 2 in which $E(v)$ is calculated by fixing $\gamma_1$as 0.05, ρ as 0.5 and varying the values of γ_2 from 0.08 to 3. By analysing the values of $E(v)$ it is possible to conclude that for each value of $C, E(v)$ increases gradually whenever the value of γ_2 nearer to as well as far away from γ_1. That is $E(v)$ remains stable with in a favourable range when the distance between the points γ_1 and γ_2 increases to a value nearer to 0.05 and then increase immediately when the respective distance overcomes that value. Hence if we choose γ_1 as 0.05 the better choice for the value of γ_2 in order to reduce the average waiting time of the system lies in a neighbourhood of 0.1 with a small radius. Similarly the results indicate that if we select γ_1 as 0.07 the suitable value for γ_2 lies in a small neighbourhood of 0.4. Accordingly for each choice of γ_1 there exists an apt choice for γ_2 such that the average waiting time corresponding to this particular pair will offer the minimum waiting time for the customers in the system.

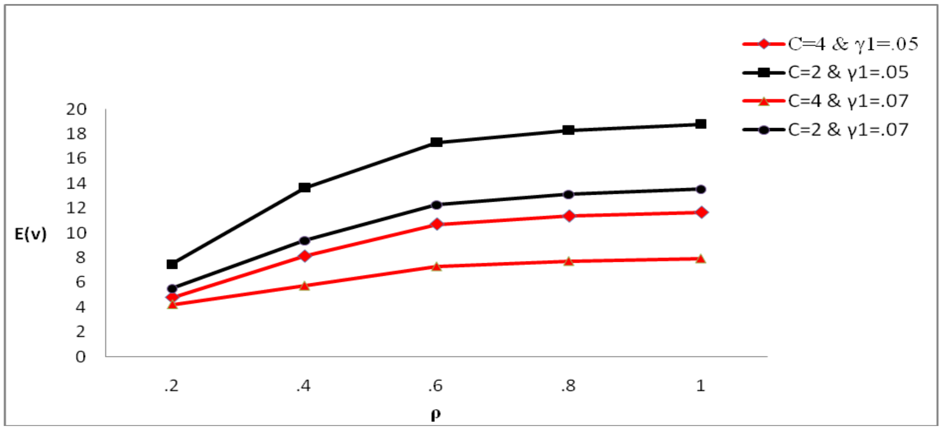

Fig. 2 Mean time in the system by varying ρ for $\gamma_2 = 1$

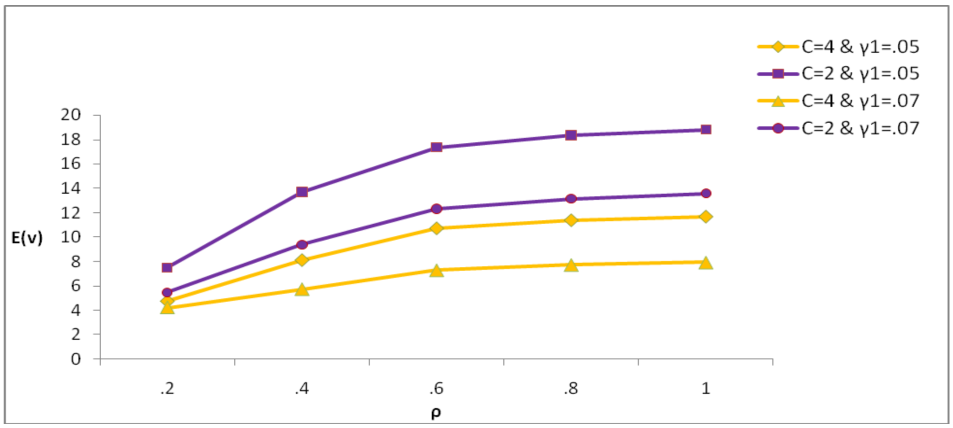

Fig.3 Mean time in the system by varying ρ for $\gamma_2 = 2$

Now for the second case let us assume that $\gamma_1 \geq \gamma_2$. Fix γ_1 as 0.05 and consider the values 0.02 and 0.04 for γ_2. By analysing the graphs which show the variations of $E(v)$ against ρ given in figure 4, we reached in a conclusion that $E(v)$ increases as $1/\gamma_2$ increases for each value of C. Figure 5 shows the results for γ_1= 0.07 and γ_2 = 0.02 & 0.04 then we obtained the same trend as above. Hence in this case the average waiting time is more sensitive to the mean duration of the vacation which is taken after a zero working period. Further we observe that by increasing the number of servers we can extend the relaxation time of them without increasing the waiting time of the system noticeably. The proposed model with the parameter relation $\gamma_1 \geq \gamma_2$ can be widely introduced in some manufacturing sectors mainly in companies using machines which perform with only high watts of electricity. As a part of preserving electricity and to increase the life span of those machines let us introduce breaks of different durations for these machines. It should be ensured that the duration of the breaks given for the respective machines in the absence of objects to proceed is much higher than that of the casual break given for them. If we scheduled the working period of the machines likewise with suitable choices for the durations of the respective vacations, the system will provide better outputs by saving electricity to an extent.

The whole analysis enables us to recognize that the proposed model with the parameter relation $\gamma_2 \leq \gamma_1$ is more preferable to introduce in most of the real life situations. Since in the respective case the expected waiting time is comparatively controllable. As a result more relaxation can afford for the servers and hence they will provide better come fast service when they are in active mode.

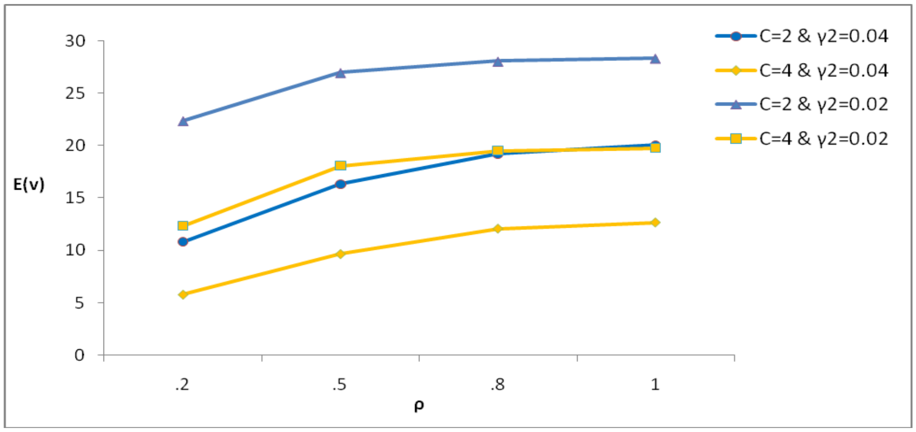

Fig. 4 Mean time in the system by varying ρ for γ_1 = 0.05

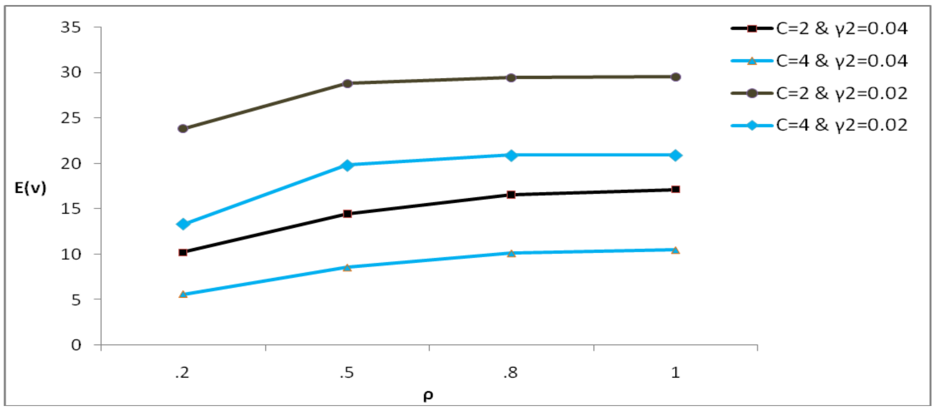

Fig. 5 Mean time in the system by varying ρ for γ_1 = 0.07

IV. CONCLUSIONS

In this paper we proposed a multi server queueing system with differentiated non working vacations. We derived the formulas for calculating the average waiting time and steady state probabilities of the recommended model. While investigating about the influences of durations of vacations in the value of average waiting time we obtained that the average waiting time is more responsive to the mean duration of the vacation, which is to be taken as the highest in value than the other. It is quite trivial that when the number of servers increases in a queueing system, the average waiting times will increases slowly. But in our proposed queueing model this fact can be taken as an additional advantage for the servers in the sense that by increasing the number of servers in the queueing system we can enlarge the period of vacations offered for them. Hence within the queueing system servers become more relaxed and as a result the system will provide better service within a limited time by maintaining the average waiting time of the customers with in a favourable range. The suggested model can be applied in various real life situations in order to avoid the stress of the service providers. Also the model can be recommended for emerging projects of various streams of business and industries.

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