FORMATION OF SPECIAL DIO-QUADRUPLE INVOLVING PRONIC NUMBER WITH PROPERTY D (5)

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***Abstract*— In this communication, the scrutiny of forming a special Dio-quadruples (a,b,c_0,c_1) such that the multiplication of any two members of the set subtracted by twice their sum and increased by the number five is a perfect square.**

***Keywords*— Pronic number, Diophantine quadruples, Pell equation, Integer solutions, Diophantine triples.**

***Notation*—** $\mathrm{Pro_n} = \text{Pronic number of rank n.}$

I. Introduction

Many mathematicians considered the problem of the existence of a Diophantine quadruples with property $D(n)$ for any arbitrary integer n [1] and also for any linear polynomials in n. Further, various authors considered the connections of the problem of Diaphanous, Davenport and Fibonacci numbers in [2-8].

In this communication, we construct special Dio-quadruples where the multiplication of any two members of the quadruples subtracted by twice their sum and increased by the number five is a perfect square.

II. Method of analysis

Let $a = 2(\mathrm{Pro_n} + 1)$ and $b = 2(\mathrm{Pro_{n-1}} + 1)$ be two different integers such that $ab - 2(a+b) + 5$ is a perfect square.

Let c_k be any non-zero integer such that

$$ac_k - 2(a + c_k) + 5 = u_k^2 \tag{1}$$

$$bc_k - 2(b + c_k) + 5 = v_k^2 \tag{2}$$

Terminating c_k between (1) and (2), we get

$$(b - a) = (b-2)u_k^2 - (a-2)v_k^2 \tag{3}$$

Considering, the linear transformations

$$u_k = x_k + (a-2)y_k \text{ and } v_k = x_k + (b-2)y_k \tag{4}$$

Substituting the values of u_k and v_k in (3), we get

$$x_k^2 = (2n^2 + 2n)(2n^2 - 2n)y_k^2 + 1 \tag{5}$$

The above equation is nothing but the Pell equation whose general solution is given by

$$\left.\begin{aligned} x_k &= \frac{1}{2}\left[\left(2n^2 - 1 + \sqrt{4n^4 - 4n^2}\right)^{k+1} + \left(2n^2 - 1 - \sqrt{4n^4 - 4n^2}\right)^{k+1}\right] \\ y_k &= \frac{1}{2\sqrt{4n^4 - 4n^2}}\left[\left(2n^2 - 1 + \sqrt{4n^4 - 4n^2}\right)^{k+1} - \left(2n^2 - 1 - \sqrt{4n^4 - 4n^2}\right)^{k+1}\right] \end{aligned}\right\} \tag{6}$$

Put $k=0$ in (6), and substituting the values of x_0 and y_0 in (4), we get

$$u_0 = 4n^2 + 2n - 1$$

substituting u_0 in (1), we get

$$c_0 = 8n^2 = 2(\text{Pro}_{2n} - 2n)$$

put $k=1$ and applying the same procedure mentioned above, we get

$$c_1 = 128n^6 - 160n^4 + 56n^2 - 2 = 2(\text{Pro}_{8n^3} - 80n^4 - 8n^3 + 28n^2 - 1)$$

Thus, we attain $\{2(\text{Pro}_n + 1),\ 2(\text{Pro}_{n-1} + 1),\ 2(\text{Pro}_{2n} - 2n),\ 2(\text{Pro}_{8n^3} - 80n^4 - 8n^3 + 28n^2 - 1)\}$ as a special Dio-quadruple with the property D (5).

Some of the numerical examples of the above special Dio-quadruples are presented below.

TABLE I
NUMERICAL EXAMPLES

n	(a, b, c_0, c_1)
1	(6,2,8,22)
2	(14,6,32,5854)
3	(26,14,72,80854)
4	(42,26,128,484222)
5	(62,42,200,1901398)

A. Remarkable Observation

For the choice of (a, c_0) and (b, c_0) and applying the same procedure, we acquire special Dio-quadruples with the property D (5) and given in the following table.

TABLE 2
SOME EXAMPLES OF SPECIAL DIO-QUADRUPLES

a	b	c_0	c_1
$2(\text{Pro}_n + 1)$	$2(\text{Pro}_{2n} - 2n)$	$2(\text{Pro}_{3n} - 1)$	$2(\text{Pro}_{24n^3} + 768n^5 - 80n^4 - 344n^3 + 12n^2 + 40n - 5)$
$2(\text{Pro}_{n-1} + 1)$	$2(\text{Pro}_{2n} - 2n)$	$2(\text{Pro}_{3n} - 6n - 1)$	$2(\text{Pro}_{24n^3} - 768n^5 - 80n^4 + 296n^3 + 12n^2 - 40n - 5)$

III. CONCLUSION

In this paper, we construct the special Dio-quadruples using Pronic numbers with the property D (5). One may seek for other special Dio-quadruples made up of different numbers with suitable properties.

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