Sagdeev Potential Approach to Study the Dust Acoustic Waves in Dusty Plasma with Nonthermal Ions

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***Abstract*-Nonlinear dust acoustic waves (DAWs) in warm dusty plasma with variable dust charge and dust pressure in presence of nonthermal ions and Boltzman distributed electrons are investigated. Sagdeev potential equation is derived by nonperturbative approach. Small amplitude solitary waves are studied by obtaining the *sech* solution of the S.P.equation. Possible ranges of the parameters for the existence of solitary waves are obtained numerically.**

Keywords- **Nonthermal ions, Sagdeev potential equation, sec-h solution**

I. Introduction

Nonlinear phenomena of dusty plasma have been growing most rapidly in the field of plasma dynamics. The presence of dust grains in plasma gives rise to a new scope of research and helps in solving many astrophysical and other scientific problems. Dust particles are very small in size, whose diameters are measured in micrometers and even sub micrometers. Though dust particles are static due to their heavy mass and have no charge, but due to the collision with other charged particles viz. ions, electrons etc., they become charged and start oscillation. The oscillatory behavior dust particles are being studied by means of some waves viz. shock waves, solitary waves, sheath, double layer etc. The theoretical features of dusty plasma and their applications have been observed in the Earth's magnetosphere, comentary tail, planetary rings etc., which makes the branch most interesting [1-2]. The first theoretical investigation on dusty plasma had been done by Rao et.al [3] in 1990. He reported the existence of DAW of low frequency in unmagnetized plasma. Later, the findings of Rao *et.al* were experimentally verified by Barkan *et.al* [4]. Dust ion acoustic waves (DIAWs) at higher frequency were studied by Shukla and Silin [5].

Many researchers have been done so far on DAW in different physical situations under the influence of other components. Labany *et.al* [6] had studied DAW in dusty plasma in presence of isothermal electrons and two temperature ions. The effect of nonthermal ions and electrons can also influence the nature of both linear and nonlinear behavior of solitary waves. Ciarns [7] studied ion acoustic waves in presence of nonthermal electrons and found similar structures as observed by Freja and Viking satellites [8]. Large amplitude dust acoustic solitary waves with finite dust temperature was investigated by K. Annou and R. Annou [9] in an unmagnetized dusty plasma consisting of inertial charged dust grains, Boltzman electrons and nonthermal ions. Rahman and Mamun [10] had also studied on nonlinear propagation of dust acoustic solitary waves with arbitrarily charged dust and trapped electrons.

In this work, we have considered a fluid model of plasma consisting of negatively charged dust, Boltzman distributed electrons and nonthermal ions and investigated the influence of phase speed (Mach number), concentration of nonthermal ions, relative densities, relative temperatures on the amplitude and width of small amplitude solitary waves.

II. BASIC EQUATIONS

We have considered unmagnetized collisionless three components dusty plasma governing by the fluid equations of continuity and equation of motion supplemented by pressure equation of dust. For the charged particles, the nonthermal distribution of ions and Boltzman distribution of electrons are followed by the Poisson's equation. The normalized equations are as follows:

$$\frac{\partial n_d}{\partial t}+\frac{\partial\left(n_d v_d\right)}{\partial x}=0 \tag{1}$$

$$\frac{\partial v_d}{\partial t}+v_d\frac{\partial v_d}{\partial x}+\frac{\sigma_d}{n_d}\frac{\partial p_d}{\partial x}=\frac{\partial\phi}{\partial x} \tag{2}$$

$$\frac{\partial p_d}{\partial t}+v_d\frac{\partial p_d}{\partial x}+3p_d\frac{\partial v_d}{\partial x}=0 \tag{3}$$

$$\frac{\partial^2\phi}{\partial x^2}=n_d+\mu n_e-\left(1+\mu\right)n_i \tag{4}$$

$$n_e=\exp\left(\sigma_e\phi\right) \tag{5}$$

$$n_i=\left(1+\beta\phi+\beta\phi^2\right)\exp\left(\phi\right) \tag{6}$$

where, $\sigma_d=\frac{T_d}{T_{eff}}$, $\sigma_e=\frac{T_{eff}}{T_e}$, $\mu=\frac{n_{e0}}{z_d n_{d0}}$, $T_{eff}=\left[\frac{1}{z_d n_{d0}}\left(\frac{n_{e0}}{T_e}+\frac{n_{i0}}{T_i}\right)\right]^{-1}$, $\beta=\frac{4\alpha}{1+3\alpha}$, α being the population of nonthermal ions.

We have normalized v_d by effective dust acoustic speed $c_d=\sqrt{\frac{z_d T_{eff}}{m_d}}$, p_d by $z_d n_{d0} T_d$, ϕ by $\frac{T_{eff}}{e}$, n_j by n_{j0} where $j=d,e,i$, time t and space variable x by dust plasma period $\omega_{pd}^{-1}=\sqrt{\frac{m_d}{4\pi e^2 z_d^{\,2} n_{d0}}}$ and Debye length $\lambda_D=\sqrt{\frac{T_{eff}}{4\pi e^2 z_d n_{d0}}}$ respectively.

III. DERIVATION OF SAGDEEV POTENTIAL EQUATION

To derive the Sagdeev potential equation from the above set of basic equations, we consider the transformation $\xi = x - Mt$, where M is the wave Mach number or the phase speed of dust, so that the partial derivative operators with respect to space and time variables reduce to a single ordinary derivative operator as,

$$\frac{\partial}{\partial x} \equiv \frac{d}{d\xi}, \frac{\partial}{\partial t} \equiv -M\frac{d}{d\xi}.$$

Applying this transformation in (1), (2) and (3), we get

$$v_d = M\left(1 - \frac{1}{n_d}\right) \tag{7}$$

$$p_d = n_d^{\,3} \tag{8}$$

$$n_d^{\,2} = \frac{\frac{M^2}{2} + \frac{3\sigma_d}{2} + \phi \pm \sqrt{\left(\frac{M^2}{2} + \frac{3\sigma_d}{2} + \phi\right)^2 - 3\sigma_d M^2}}{3\sigma_d} \tag{9}$$

Let $n_d = \sqrt{p} - \sqrt{q}$

$$\therefore n_d^{\,2} = p + q - 2\sqrt{pq} \tag{10}$$

Comparing (9) and (10),

$$p + q = \frac{M^2 + 3\sigma_d + 2\phi}{6\sigma_d}$$

$$pq = \frac{\left(M^2 + 3\sigma_d + 2\phi\right)^2 - 12\sigma_d M^2}{144\sigma_d^{\,2}}$$

These give

$$p = \frac{\left(M + \sqrt{3\sigma_d}\right)^2 + 2\phi}{12\sigma_d}$$

$$q=\frac{\left(M-\sqrt{3\sigma_d}\right)^2+2\phi}{12\sigma_d}$$

From (10) we get,

$$n_d=\frac{1}{2\sqrt{3\sigma_d}}\left[\left\{\left(M+\sqrt{3\sigma_d}\right)^2+2\phi\right\}^{\frac{1}{2}}-\left\{\left(M-\sqrt{3\sigma_d}\right)^2+2\phi\right\}^{\frac{1}{2}}\right] \quad (11)$$

Using (5), (6) and (11) in (4) we get,

$$\frac{d}{d\xi}\left[\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2\right]=\left\{n_d+\mu n_e-(1+\mu)n_i\right\}\frac{d\phi}{d\xi} \quad (12)$$

Integrating with respect to ξ with boundary conditions $\frac{d\phi}{d\xi}\to 0, \phi\to 0$ as $|\xi|\to\infty$ we get,

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2+S(\phi)=0 \quad (13)$$

where,

$$S(\phi)=\frac{1}{6\sqrt{3\sigma_d}}\left(\left[\left\{\left(M-\sqrt{3\sigma_d}\right)^2+2\phi\right\}^{\frac{3}{2}}-\left(M-\sqrt{3\sigma_d}\right)^3\right]-\left[\left\{\left(M+\sqrt{3\sigma_d}\right)^2+2\phi\right\}^{\frac{3}{2}}-\left(M+\sqrt{3\sigma_d}\right)^3\right]\right)$$

$$+\frac{\mu}{\sigma_e}\left\{1-\exp\left(\sigma_e\phi\right)\right\}+(1+\mu)\left\{\left(1+\beta-\beta\phi+\beta\phi^2\right)\exp(\phi)-(1+\beta)\right\}$$

The equation (13) is called the Sagdeev potential equation. This is an energy integral and describes the motion of the oscillatory dust particles within a potential well ϕ with velocity $\frac{d\phi}{d\xi}$ at instant ξ. The term $S(\phi)$ is a pseudopotential, called Sagdeev potential and represents the energy of the wave with amplitude ϕ.

IV. CONDITIONS FOR EXISTENCE OF SOLITARY WAVES

In order to exist the solitary wave solution of the equation (13), the pseudopotential $S(\phi)$ should satisfy the following conditions:

(i) $S(\phi)=0$ at $\phi=0$ and $\phi=\phi_m$, where ϕ_m is the maximum value of ϕ and $S(\phi)$ is negative between $\phi=0$ and $\phi=\phi_m$

(ii) $S'(\phi)=0$ at $\phi=0$

Here, ϕ represents the amplitude of the solitary waves. The waves are said to be compressive for positive values of ϕ. For negative values of ϕ, the waves are called rarefactive. The above conditions are verified and the compressive solitary waves are found to exist for certain values of the parameters.

V. SOLUTION FOR SMALL AMPLITUDE SOLITONS

We can express the pseudopotential as,

$$S(\phi)=a_1\phi^2+a_2\phi^3+a_3\phi^4+a_4\phi^5+......................$$

Considering up to 3rd degree term, the Sagdeev potential equation (13) becomes,

$$\left(\frac{d\phi}{d\xi}\right)^2=b_1\phi^2-b_2\phi^3 \tag{14}$$

where,

$$b_1=\frac{1}{2\sqrt{3\sigma_d}}\left(b^{-1}-a^{-1}\right)-\left\{(1+\mu)(1+\beta)-\mu\sigma_e\right\}$$

$$b_2=\frac{1}{6\sqrt{3\sigma_d}}\left(b^{-3}-a^{-3}\right)+\frac{1}{3}\left\{(1+\mu)(1+4\beta)-\mu\sigma_e{}^2\right\}$$

The solution of (14) is given by,

$$\phi=\phi_0\sec h^2(\Delta\xi) \tag{15}$$

where, $\phi_0=\frac{b_1}{b_2}$ is the amplitude and $\Delta^{-1}=\frac{2}{\sqrt{b_1}}$ is the width of the solitary wave.

VI. RESULTS AND DISCUSSIONS

To verify the conditions for existence of solitary waves, we plot $S(\phi)$ vs. ϕ for certain values of the parameters. It has been observed in figure 1, that compressive solitary wave for arbitrary amplitude exist and thus indicating that the conditions are being satisfied.

The small amplitude solitary waves are studied with the help of (15) by plotting ϕ vs. ξ. The variation of amplitudes and widths are investigated by assigning different values of the parameters.

In figure 2, we have used three different values of α (the population of nonthermal ions) as 0.40, 0.42 and 0.45 keeping the values of the other parameters fixed and observed that the amplitude is going decreasing with the increased values of α. However, no significant changes in width are observed due to the change of α.

The effect of Mach number (phase speed of dust acoustic waves) is studied by varying the Mach number as $M =$0.68, 0.69, and 0.70. We have seen in figure 3, that the amplitudes of the solitons are increasing as the Mach numbers are going increasing but the widths are decreasing. Thus, the phase speed of the acoustic wave plays an important role in its propagation, and there is possibility to become the wave spiky once the phase speed reaches the maximum value (>1).

The number of dust charged particles also plays a direct role in the propagation of nonlinear dust acoustic waves. In figure 4, we have taken $\mu =$0.01, 0.02, and 0.03 and observed that the amplitude and width both are decreasing while μ is increasing. As the number of dust charged particles increases, the density ratio $\mu\left(=\frac{n_{e0}}{z_d n_{d0}}\right)$ decreases, so both amplitude and width are increased if the number of dust charged particles increases.

In Figure 5(a) and 5(b), we have studied the effect of temperatures of dust and electrons. In figure 5(a) we consider the three values of σ_e as 0.5, 0.9 and 1.3 and observed that the amplitude and width both increases when σ_e increases. Since $\sigma_e = \frac{T_{eff}}{Te}$, so effective temperature increases when σ_e increases. In figure 5(b), it is observed that the amplitudes are going decreasing, when the values of σ_d (= 0.7, 0.8 and 0.9) are increasing, but at the same time widths are increasing. Since, $\sigma_d = \frac{T_d}{T_{eff}}$, so σ_d decreases as the effective temperature increases. Thus, from figure 5(a) and 5(b), we have seen that, the amplitude of the solitons increases, if the effective temperature increases. Also the width of the solitons increases, when the dust temperature increases and electron temperature decreases.

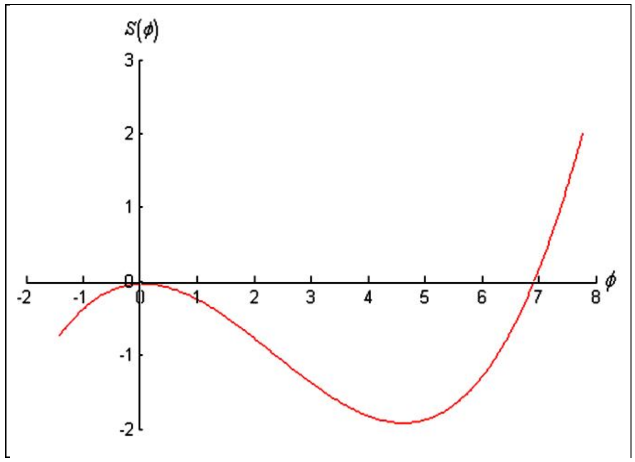

Fig. 1: $S(\phi)$ vs. ϕ for $\sigma_d = 0.3, \sigma_e = 0.1, M = 0.7, \mu = 0.01, \alpha = 0.40$

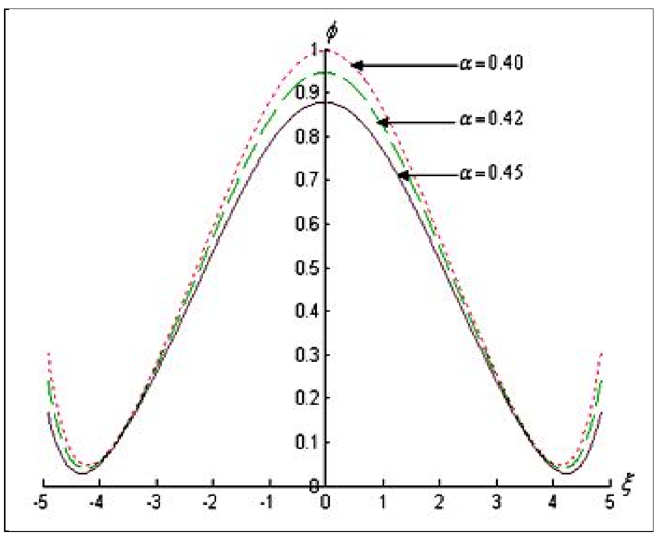

Fig. 2: ϕ vs. ξ for $\mu = 0.01, \sigma_d = 0.3, \sigma_e = 0.9, M = 0.7$ and $\alpha = 0.40, 0.42, 04.5$

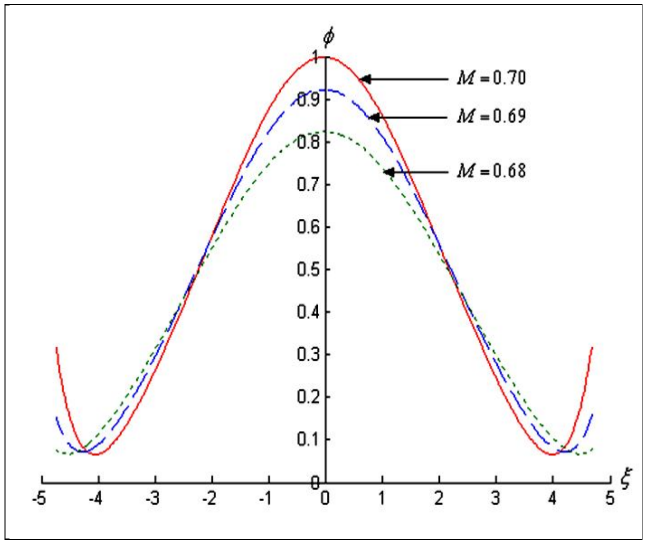

Fig. 3: ϕ vs. ξ for $\mu = 0.01, \sigma_d = 0.3, \sigma_e = 0.9, \alpha = 0.40$ and $M = 0.68, 0.69, 0.70$

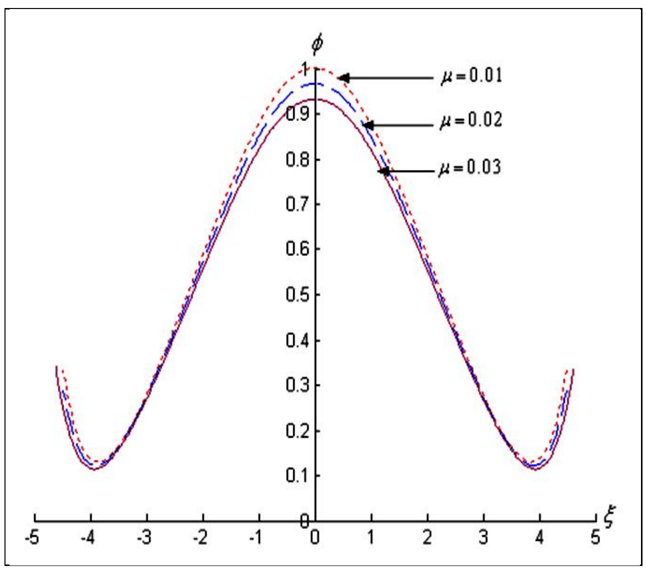

Fig. 4: ϕ vs. ξ for $\sigma_d = 0.3, \sigma_e = 0.9, \alpha = 0.40, M = 0.68$ and $\mu = 0.01, 0.02, 0.03$

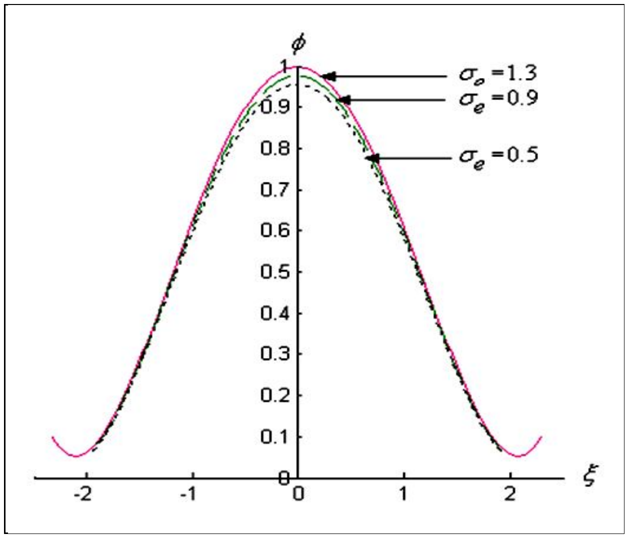

Fig. 5(a) : ϕ vs. ξ for $\mu = 0.01, \sigma_d = 0.3, \alpha = 0.40, M = 0.68$ and $\sigma_e = 0.5, 0.9, 1.3$

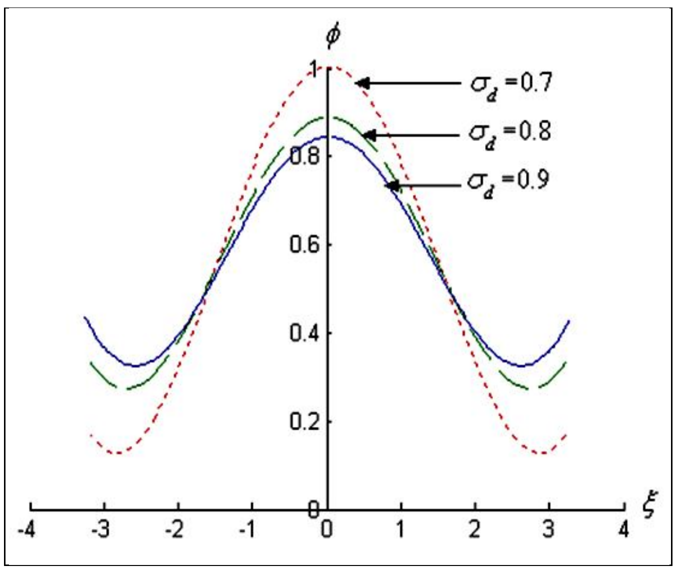

Fig. 5(b) : ϕ vs. ξ for $\mu = 0.01, \sigma_e = 0.9, \alpha = 0.40, M = 0.68$ and $\sigma_d = 0.7, 0.8, 0.9$

VII. CONCLUSIONS

The main objective of this work is to study the propagation of nonlinear dust acoustic solitary waves in warm dusty plasma in presence of nonthermal ions through nonperturbative approach. In this multicomponent plasma, we have considered negatively charged dust particles, Boltzman distributed electrons and nonthermal ions. By applying nonperturbative approach, the Sagdeev potential equation is derived and solved for sec-h solution. The conditions for existence of solitary wave for arbitrary amplitude are satisfied. The small amplitude waves are studied and the variation of amplitudes and widths are investigated for different values of the parameters. As a future scope of study, our present work could be of motivation to study the other nonlinear behaviors such as formation of sheath, double layer etc. in any multicomponent plasma.

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