VAGUE SEMI-OPEN SETS AND VAGUE SEMI-CONTINUOUS FUNCTIONS IN VAGUE BITOPOLOGICAL SPACES

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Abstract— **This work defines few properties of Vague - τ_1 τ_2 - semi open sets(semi closed sets) in a Vague bitopological space .Various results of Vague - τ_1 τ_2 - semi open sets (closed sets) and Vague - τ_1 τ_2 -semi continuous in Vague bitopological spaces are also investigated.**

Keywords— **Vague topological space ,Vague bitopological space,V - τ_1 τ_2 - semi open sets, V- τ_1 τ_2 - semi closed sets ,V- τ_1 τ_2 - semi continuous function.**

I. INTRODUCTION

J.C.Kelly [5] originated the Bitopological spaces as a genuine framework by learning quasimetrics plus its conjugate .He also came out with several properties of separation to bitopological spaces,by finding few essential results.The introduction of semi-open sets and semi-continuous functions in bitopological spaces was given by S.N.Maheshwari and R.Prasad [7].Later B.Bhattacharya and A.Paul [3] introduced a new approach of γ open sets in bitopological spaces.F.H.Khedr[6] studied the properties of - τ_1 τ_2-δ - open sets .M.Arunmaran and KangeyanathanKannan[2] introduced some properties of - $\tau_1\tau_2$-δ-semi open sets/closed sets in bitopological spaces in 2017.

In this paper we study the τ_1 τ_2- semi open sets and their continuity in Vague bitopological spaces and we have provided examples for validating the theory.

II. PRELIMINARIES

Definition 2.1[9]

A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A: U \to [0,1]$ and

(ii) A false membership function $f_A: U \to [0,1]$ where t_A is the lower bound on the grade of membership of x derived from the “evidence for x” , f_A is the lower bound on the negation of x derived from the “evidence against x” , and $t_A + f_A \leq 1$. Thus the grade of membership of μ is the vague set A is bounded by a subinterval $[t_A, 1 - f_A]$ of [0,1]. This indicates that if the grade of membership of x is $\mu(x)$ then, $t_A(x) \leq \mu(x) \leq 1 - f_A(x)$.

Definition:2.2 [9]

Let A and B be vague sets of the form,

$$A = \{< x, [t_A(x), 1 - f_A(x)] > / x \epsilon X\} \text{and}$$
$$B = \{< x, [t_B(x), 1 - f_B(x)] > / x \epsilon X\}. \text{Then}$$

(i) A ⊆ B if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$ for all $x \epsilon$X.

(ii) A=B if and only if A ⊆ B and B ⊆A.
(iii) $A^c = \{< x, [f_A(x), 1 - t_A(x)] > / x \epsilon X\}$
(iv) $A \cap B = \{< x, min[t_A(x)\, t_B(x)], min[1 - f_A(x), 1 - f_B(x)] > / x \epsilon X\}$
(v) $A \cup B = \{< x, max[t_A(x)\, t_B(x)], max[1 - f_A(x), 1 - f_B(x)] > / x \epsilon X\}$.

Definition: 2.3[9]

A vague topology on X is a family τ of vague sets in X satisfying the following axioms.

(i) 0,1 ϵ τ.
(ii) $G_1 \cap G_2$ for any G_1, G_2 ϵ τ.
(iii) $\cup(G_i)$ ϵ τ for any family { G_i: i ϵ J }⊆ τ.

Definition:2.4[9]

(X, τ) is a vague topological space and let A = $\{x, t_A(x), 1 - f_A(x)\}$ be a vague set in X. Then the vague interior and a vague closure are defined by

V int (A)= ∪{G/G is a VOS in X and G⊆ A}
V cl (A)= ∩{K/K is a VCS in X and A⊆ K}

For any vague set A in (X, τ), we have Vcl (A^c)= (V int (A)) ^{c}.

Definition:2.5[2]

A non-empty set X with two (distinct) topologies τ_1 and τ_2 is called a bitopological space and it is denoted by (X, τ_1, τ_2).

Definition:2.6[2]

Assume A to be a subset of (X, τ_1, τ_2).Then, A is known as open, if A∈ $\tau_1 \cap \tau_2$. In (X, τ_1, τ_2),complement of open set is called closed set.

Definition:2.7[2]

Assume A to be a subset of (X, τ_1, τ_2). Then, A is known as τ_1 τ_2-open, if A ∈ $\tau_1 \cup \tau_2$. In (X, τ_1, τ_2), complement of τ_1 τ_2-open set is called τ_1 τ_2-closed set.

Definition:2.8[2]

Assume A to be a subset of (X, τ_1, τ_2). Then, A is known as τ_1 τ_2-semi open, if A ⊆ τ_2-cl(τ_1-int(A)).

III. PROPERTIES OF VAGUE SEMI OPEN SETS AND SEMI CLOSED SETS IN VAGUE BITOPOLOGICAL SPACES

Definition 3.1

Let X be a non-empty set. Let τ_1 and τ_2 be two vague topologies on X. Then the triplet (X, τ_1, τ_2) is called a Vague bitopological space.

Example 3.2

Let X={a,b}, τ_1={0, R_1,1}, τ_2={0, R_2,1}, where R_1={<[0.3,0.6],[0.2,0.7]},R_2={<[0.1,0.2],[0.1,0.2] >} and let A={<[0.3,0.5],[0.2,0.6] >}.

Here $\tau_1 \cup \tau_2$ ={0, R_1,1}are the vague - τ_1 τ_2- open sets and the complement of vague -τ_1 τ_2- open sets are the vague -τ_1 τ_2- closed sets respectively.

Definition 3.3

Let A be a subset of a vague bitopological space (X, τ_1, τ_2).Then A is called

(i) V - τ_1 τ_2 - semi open, if A⊆ V-τ_2 -cl(V-τ_1-int (A)).

(ii)V - τ_1 τ_2 - pre open,if A⊆ V-τ_1-int(V-τ_2-cl (A)).

(iii)V - τ_1 τ_2 - α open,if A⊆ V- τ_1-int(V-τ_2-cl(V- τ_1-int(A)).

(iv)V - τ_1 τ_2 - regular open, if A= V-τ_1 -int(V-τ_2-cl (A)).

Theorem 3.4

Every V - τ_1- closed set is V - τ_1 τ_2 - semi closed set in (X, τ_1, τ_2).

Proof:

Let A be a V - τ_1- closed set.Then A= V-τ_1 -cl(A).But V-τ_2 -int(A)= V-τ_2 -int(V-τ_1 -cl(A)) ⊆ A. Hence A is a V - τ_1 τ_2 - semi closed set in (X, τ_1, τ_2).

Remark 3.5

The converse part of the above theorem need not be true.

Theorem 3.6

Let A be a subset of a vague bitopological space (X, τ_1, τ_2).Then A is a V - τ_1 τ_2 - semi closed iff A ⊇ V-τ_2 -int(V-τ_1-cl (A)).

Proof:

Let A⊇ V-τ_2 -int(τ_1-cl (A)) .Then there exists a V-τ_1- closed set (=V-τ_1 -cl(A)) such that V-τ_2 -int(U) ⊆ A⊆U.Thus A is a V - τ_1 τ_2 - semi closed set in (X, τ_1, τ_2).

Conversely,Let A be a V - τ_1 τ_2 - semi closed set.Let U be a V - τ_1- closed set .Then V-τ_2 -int(U) ⊆ A⊆U.Thus , V-τ_2 -int(V-τ_1-cl (A)) ⊆ A.

Theorem 3.7

If A and B are V - τ_1 τ_2 – semi open sets in a vague bitopological space(X, τ_1, τ_2).Then A∪B is also a V - τ_1 τ_2 – semi open set.

Proof:

Since A and B are the V - τ_1 τ_2 - semi open sets, A⊆V-τ_2 -cl(V-τ_1-int (A)) and B⊆V-τ_2 -cl(V-τ_1-int (B)) .This implies,

Jhj A∪B ⊆ V-τ_2 -cl(V-τ_1-int (A)) ∪V-τ_2 -cl(V-τ_1-int (B)).

So A∪B ⊆V-τ_2 -cl(V-τ_1-int (A∪B)).Hence the result follows.

Theorem 3.8

If A and B are V – τ_2 τ_1 - semi open sets in a vague bitopological space(X, τ_1, τ_2).Then A∪B is also a V – τ_2 τ_1 - semi open set.

Proof: Since A and B are the V - τ_2 τ_1 - semi open sets, V-τ_1 -cl(V-τ_2-int (A))⊇A and V-τ_1 -cl(V-τ_2-int (B)) ⊇B.This implies,

V-τ_1 -cl(V-τ_2-int (A)) ∪ V-τ_1 -cl(V-τ_2-int (B)) ⊇ A∪B .

So V-τ_1-cl(V-τ_2-int (A∪B)))) ⊇ A∪B. Hence the result .

Remark 3.9

If A and B are V – τ_2 τ_1 - semi closed sets in a vague bitopological space(X, τ_1, τ_2) then ,so is A∩B.

Theorem 3.10

If A and B are $V - \tau_1\ \tau_2$ - semi closed sets in a vague bitopological space(X, τ_1, τ_2) then ,so is A∩B.

Proof:

Since A,B are $V - \tau_1\ \tau_2$ - semi closed sets,we have A ⊇V-τ_2-int(V-τ_1-cl (A)) and B ⊇ V-τ_2-int(V-τ_1-cl (B)).This implies ,

A∩B ⊇ V-τ_2-int(V-τ_1-cl (A)) ∩ V-τ_2-int(V-τ_1-cl (B)).

So, A∩B ⊇V-τ_2-int(V-τ_1-cl (A∩B)).Hence the result.

Remark 3.11

Let A and B be the subsets of a vague bitopological space(X, τ_1, τ_2) and A∪B is a $V - \tau_1\ \tau_2$ - semi open set. Then A,B need not be a $V - \tau_1\ \tau_2$ - semi open set.

Example 3.12

X={a,b},τ_1={0,R_1,1},τ_2={0,R_2,1}where

R_1={<[0.2,0.3],[0.3,0.4]>},R_2={<[0.3,0.4],[0.4,0.5] >} and let A={<[0.1,0.2],[0.2,0.4] >},

B={<[0.2,0.3],[0.3,0.4] >}.

Here the vague - $\tau_1\ \tau_2$ open sets are {0,R_2,1} and A∪B is a $V - \tau_1\ \tau_2$ - semi open set but A,B is not a $V - \tau_1\ \tau_2$ - semi open set.

Remark 3.13

Let A and B be the subsets of a vague bitopological space (X, τ_1, τ_2) and A∩B is a $V - \tau_1\ \tau_2$ - semi closed set.Then A need not be $V - \tau_1\ \tau_2$ - semi closed set.

Example 3.14

Let X= {a,b}, τ_1={0,R_1,1},τ_2={0,R_2,1}where

R_1={<[0.2,0.3],[0.3,0.4]>},R_2={<[0.3,0.4],[0.4,0.5] >} and let A={<[0.1,0.2],[0.2,0.4] >},

B={<[0.2,0.3],[0.3,0.4] >}.

Here the vague- $\tau_1\ \tau_2$ open set is {0,G_2,1} and A∩B={<[0.1,0.2],[0.2,0.4] >} is a $V - \tau_1\ \tau_2$ - semi closed set.B is also a $V - \tau_1\ \tau_2$ - semi closed set whereas A is not a $V - \tau_1\ \tau_2$ - semi closed set, since A does not contain {<[1]>}.

IV. PROPERTIES OF VAGUE- $V - \tau_1\ \tau_2$– SEMI CONTINUOUS FUNCTIONS

Definition 4.1[2]

Assume (X, τ_1, τ_2) and (Y, σ_1, σ_2) as two vague bitopological spaces.A function f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$is called a vague- $\tau_1\ \tau_2$ – continuous ,if an inverse image of each vague- σ_1 open set in Y is a vague - $\tau_1\tau_2$ - open set in X.

Example 4.2

Assume X=Y={a,b}, τ_1={0,R_1,1},τ_2={0,R_2,1}, σ_1={0,R_3,1}, σ_2={0,R_4,1}where

R_1= {<[0.3,0.5],[0.6,0.7]>}, R_2 = {<[0.2,0.3],[0.3,0.4]>}, R_3 = {<[0.3,0.5],[0.6,0.7]>},

R_4 = {<[0.2,0.3],[0.3,0.4]>}.

Now the $V - \tau_1\ \tau_2$ -open sets are {0,R_1,1}.If f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ defined as f(0)=0,f(a)=a,f(1)=1,then f is a vague- $\tau_1\ \tau_2$- continuous .

Example 4.3

Assume X=Y={a,b}, τ_1={0,R_1,1},τ_2={0,R_2,1}, σ_1={0,R_3,1}, σ_2={0,R_4,1}where

R_1= {<[0.3,0.6],[0.6,0.8]>}, R_2 = {<[0.2,0.3],[0.4,0.5]>}, R_3 = {<[0.3,0.6],[0.6,0.8]>},

R_4 = {<[0.1,0.2],[0.2,0.3]>}.

Now the V – τ_1 τ_2 -open sets are {0,R_1,1}.If f: (X, τ_1, τ_2)→ (Y, σ_1, σ_2) defined as f(0)=0,f(a)=a,f(1)=1,then f is a vague - τ_1 τ_2 continuous .

Definition 4.4

Assume (X, τ_1, τ_2) and (Y, σ_1, σ_2) as two vague bitopological spaces. A function f: (X, τ_1, τ_2)→ (Y, σ_1, σ_2) is called a vague- τ_1 τ_2 –semi continuous ,if the inverse image of V is a vague - τ_1 τ_2 - semi open set in X,for every vague- σ_1 open set V in Y.

Example 4.5

Assume X=Y={a,b}, τ_1={0,R_1,1},τ_2={0,R_2,1}, σ_1={0,R_3,1}, σ_2={0,R_4,1}where R_1= {<[0.3,0.6],[0.6,0.8]>}, R_2 = {<[0.2,0.3],[0.4,0.5]>}, R_3 = {<[0.3,0.6],[0.6,0.8]>}, R_4 = {<[0.2,0.3],[0.4,0.5]>}.

Here vague - σ_1 open set in Y is σ_1={0,R_3,1} and each vague - σ_1 open set in Y is a vague - τ_1 τ_2- semi open set in X. Hence it is semi continuous.

Example 4.6

Assume X=Y={a,b}, τ_1={0,R_1,1},τ_2={0,R_2,1}, σ_1={0,R_3,1}, σ_2={0,R_4,1}where R_1= {<[0.4,0.6],[0.7,0.9]>}, R_2 = {<[0.1,0.3],[0.3,0.6]>}, R_3 = {<[0.4,0.6],[0.7,0.9]>}, R_4 = {<[0.2,0.4],[0.6,0.8]>}.

Here vague - σ_1 open set in Y is σ_1={0,R_3,1} and we find that each vague - σ_1 open set in Y is a vague - τ_1 τ_2 semi open set in X. Hence it is semi continuous.

Remark 4.7

If f: (X, τ_1, τ_2)→ (Y, σ_1, σ_2) and g: (Y, σ_1, σ_2))→ (Z, η_1, η_2) be two vague- τ_1 τ_2 –semi continuous functions ,then gof: (X, τ_1, τ_2) →(Z, η_1, η_2) need not be a vague- τ_1 τ_2 –semi continuous.

Example 4.8

If X=Y=Z={a,b} and if f:X →Y, g:Y→Z, τ_1={0,R_1,1},τ_2={0,R_2,1}, σ_1={0,R_3,1}, σ_2={0,R_4,1}, η_1 = {0,R_5,1}, η_2={0,R_6,1} where R_1= {<[0.2,0.4],[0.4,0.5]>}, R_2 = {<[0.3,0.6],[0.5,0.6]>}, R_3 = {<[0.3,0.5],[0.5,0.6]>}, R_4 = {<[0.4,0.6],[0.6,0.7]>}, R_5 = {<[0.6,0.5],[0.6,0.7]>}, R_6 = {<[0.4,0.5],[0.5,0.6]>}.

We find gof need not be a vague - τ_1 τ_2 semi continuous ,while f and g are vague - τ_1 τ_2 - semi continuous function.

Example 4.9

If X=Y=Z={a,b} and if f:X →Y, g:Y→Z, τ_1={0,R_1,1},τ_2={0,R_2,1}, σ_1={0,R_3,1}, σ_2={0,R_4,1}, η_1={0,R_5,1}, η_2={0,R_6,1} where R_1= {<[0.3,0.5],[0.5,0.6]>},R_2={<[0.4,0.6],[0.3,0.4]>}, R_3={<[0.4,0.6],[0.6,0.7]>}, R_4={<[0.5,0.7],[0.7,0.8]>}, R_5={<[0.5,0.7],[0.7,0.8]>}, R_6 = {<[0.5,0.6],[0.6,0.7]>}then gof need not be a vague - τ_1 τ_2 - semi continuous function.

Here f and g are vague - τ_1 τ_2 - semi continuous. Since R_3 is not contained in V-τ_2 -cl(V-τ_1-int (R_3)) in gof ,it need not be a vague - τ_1 τ_2 - semi continuous function.

Theorem 4.10

Assume f: (X, τ_1, τ_2)→ (Y, σ_1, σ_2).Then f is a vague - τ_1 τ_2 semi continuous function iff f^{-1}(U) is a vague - τ_1 τ_2 - semi closed in X, for each vague- σ_1 closed set U in Y.

Proof:

The proof is obvious from the definition .

Theorem 4.11

Assume f: (X, τ_1, τ_2)→ (Y, σ_1, σ_2) to be a vague - τ_1 τ_2 - semi continuous,so then,for all vague- σ_1 open set V in Y,there exists a vague - τ_1 τ_2 - semi open set in X such that f(P) ⊆ V.

Proof:

Assume f to be a vague - τ_1 τ_2 - semi continuous. By theorem 4.10 , f^{-1}(U) is a vague - τ_1 τ_2 - semi closed set in X, for all vague- σ_1 closed set U in Y . Assume V to be a vague- σ_1 open set in Y. So, σ_1 closed set in Y = Y\V which implies f^{-1}(Y\V) is vague - τ_1 τ_2 - semi closed set in X. But f^{-1}(Y\V)= f^{-1}(Y)\ f^{-1}(V)=X\ f^{-1}(V) is a vague - τ_1 τ_2 - semi closed set in X. Hence f^{-1}(V) is a vague - τ_1 τ_2 - semi open in X. Assume P= f^{-1}(V).Thus, f(P)=f(f^{-1}(V)) ⊆V.

Proposition 4.12

A constant function f: (X, τ_1, τ_2)→ (Y, σ_1, σ_2) is vague - τ_1 τ_2 - semi continuous.

Proof:

Let V be a vague - σ_1 open set in Y.If c ϵ V,then X= f^{-1}(V) is a vague - τ_1 τ_2 - semi open .If c does not belong to V, then it is obvious that f^{-1}(V) = ϕ is vague - τ_1 τ_2 - semi open. As X, ϕ are vague - τ_1 τ_2 - semi open sets in X , f is vague - τ_1 τ_2 - semi continuous.

V. CONCLUSIONS

Few properties of V- τ_1 τ_2 - semi open along with V- τ_1 τ_2 - semi continuous functions in Vague bitopological spaces are discussed.We are in a plan to outstretch our work to V- τ_1 τ_2 – connectedness. We are concerned to find more outcomes in Vague bitopological spaces.

REFERENCES

[1] Ajoy Mukharjee , " *Some new bitopological notions*",Nouvelle Serie ,tome 93(107),165-172,2013.

[2] M.Arunmaran and Kangeyanathan Kannan, "*Some properties of τ_1 τ_2 -δ - semi open sets /closed sets in bitopological spaces"*,International journal of pure and applied mathematics,vol.115,no.4,pg.759-769,2017.

[3] B.Bhattacharya and A.Paul , " *A New approach of - γ - open sets in bitopological spaces"*,Gen.Math.Notes,vol.20,no.2,February 2014,pg.95-110.

[4] T.Fukutake, "*Semi open sets on bitopological spaces*",Bull.Fukuoka University.Education ,38(3) pg.1-7,(1989)

[5] J.C.Kelly , "*Bitopological spaces*",proceedings of the London Mathematical Society ,vol. 3-13,no.1,pg.71-89,1963.

[6] F.H. Khedr , " *Properties of τ_1 τ_2 -δ open sets*" ,Fasciculi Mathematics ,52,pg.65-81,2014.

[7] S.N.Maheshwari ,R.Prasad , "*Semi open sets and semi continuous functions in bitopological spaces*" ,Mathematical Not.,26,pg.29-37,1977-1978.

[8] Marcus John Saegrove , "*On bitopological spaces*",Iowa State University,Theses and Dissertations,1971.

[9] L.Maria Presenti ,I.Arockia Rani, "*Vague Generalised Alpha closed sets in topological spaces"*,International Journal of Mathematical Archive-7[8],23-29,2016.

[10] T.Noiri,M.Rajamani and M.Maheswari, "*A Decomposition of Pairwise continuity via ideals*,Bol.Soc.Paran.Mat.(3s.)v.34 1(2016),141-149.

[11] Suwarnlatha .N.Banasode and Mandakini.A.Desurkar, "*Generalised Minimal closed sets in bitopological spaces* ,Annals of Pure and Applied Mathematics ,vol.14,no.2,pg.269-276,2017.

[12] S.V.Vani ,G.Priscilla pacifica , " *New class of closed sets in bitopological spaces*",International Journal of Scientific Research in Mathematical and Statistical Sciences,vol.5,Issue-4,pg.409-413,August 2018.