Common Fixed Point Theorem for weakly compatible mapings adopting property (EA) on Dislocated S_b- Metric Spaces

Saraswathi Muthusamy[1*] , Dhivya Subramaniam[1]
[1*,1]*Department of Mathematics*
Kandaswami Kandar's College,
(Affiliated to Periyar University, Salem)
Velur, Namakkal(Dt), Tamilnadu, India
[1*]msmathsnkl@gmail.com,[1]dhivimax@gmail.com

***Abstract*— this discourse deals with common fixed point theorems for compatible and weakly compatible mappings and also peculiar outcomes on dislocated S_b-Metric Spaces.**

***Keywords*— dislocated S_b- metric Space, Common fixed point, compatible, weakly compatible, Property (E.A).**

MSC (2010): 47H10, 55M20.

I. Introduction

The typical persuasion of the metric spaces was elaborated in numerous generalizations by several mathematicians. Mustafa and Sims [8] originated a new concept G-metric space. Sedghi and others [12] developed the idea of S_b-metric space, initiated by Bakhtin [2], as an abstraction of S-metric space and also deduced some fixed point results on complete S_b-metric space. Yumnum Rohen, Dosenovic, Radenovic [14] modifies the explanation for S_b-metric space by Souayah, Mlaiki [9] and proved coupled theme in this space. In 1986, Jungck [5] instigated the compatible mappings and turned out the existence of common fixed point theorems. Sessa [13] initiated the concept of weakly commuting mappings.

Aamri, Moutawakil [1] initiated a new property which generalized the theory of non-compatible mappings. Binayak and others [3] started the part of a property (E.A) with weak compatibleness of mappings in G-metric spaces and deduced some common fixed point results.

Hitzler [4] was innovated the concept of dislocatedness in metric space. Zeyada, Hassan, Ahmed [15] developed the completeness in dislocated quasi metric spaces and unspecialized the results by Hitzler in this space. Manoj Ughade, Daheriya [7] presented several fixed point theorems in complete dislocated metric spaces and dislocated quasi metric spaces. M.Saraswathi, S.Dhivya [10] initiated the dislocatedness in S_b–metric space and tested a fixed point idea in this space under contractive conditions.

Our grail is to propose the property (E.A) and weakly compatible functions in dislocated S_b-metric space and some common fixed point theorems to a pair of compatible and weakly compatible functions.

II. Preliminaries

A. *Definition :- [10]*

Let $\mathcal{A}$ be not an empty set with a function $dS_b: \mathcal{A}^3 \to R_0^+$ satisfying

(i) $dS_b(a,d,f) > 0$ for all $a,d,f \in \mathcal{A}$ with $a \neq d \neq f$

(ii) $dS_b(a,d,f) = 0 \implies a = d = f$

(iii) $dS_b(a,d,f) = dS_b(d,f,a) = dS_b(f,a,d) = dS_b(a,f,d) = dS_b(d,a,f) = dS_b(f,d,a)$

(iv) $dS_b(a,a,d) = dS_b(d,d,a)$ $for\ all\ x,y\ \epsilon \mathcal{A}$

(v) $dS_b(a,d,f) \leq b\,[dS_b(a,a,u) + dS_b(d,d,u) + dS_b(f,f,u)]$, for all $a,d,f,u\ \epsilon\ \mathcal{A}, b \geq 1$

Then dS_b is called dislocated S_b-metric or simply dS_b–metric and $(\mathcal{A}, dS_b)$ is called dislocated S_b-metric space or simply dS_b – metric space.

B. ***Definition:- [11]***

The function $d_qS_b : \mathcal{A}^3 \rightarrow R_0^+$ where $\mathcal{A}$ is a non-empty set is called a dislocated quasi S_b-metric (simply d_qS_b – metric) if

(i) $d_qS_b(a, d, f) = 0$ *then* $a = d = f$

(ii) $d_qS_b(a, d, f) \leq b[d_qS_b(u, u, a) + d_qS_b(u, u, d) + d_qS_b(u, u, f)]$ for all $a, d, f, u \in \mathcal{A}$

Then the pair $(\mathcal{A}, d_qS_b)$ is called dislocated quasi S_b-metric space (simply d_qS_b – metric space).

C. ***Definition :-[10]***

Let the sequence $\{x_n\}$ in dS_b –metric space $(\mathcal{A}, dS_b)$ is said to be dS_b-convergent provided that for each $\epsilon > 0$, there is a number $n_0 \in I$ such that $dS_b(x_n, x_n, l) < \epsilon$ or $dS_b(l, l, x_n) < \epsilon$, $(n \geq n_o)$. We denote it as $dS_b - \lim_{n\to\infty} x_n = l$ where l is the dS_b limit point of $\{x_n\}$.

D. ***Definition :-[10]***

A sequence $\{x_n\}$ in dS_b –metric space $(\mathcal{A}, dS_b)$ is dS_b-Cauchy if for given $\epsilon > 0$, there is a number $n_0 \in I$ such that $dS_b(x_n, x_m, x_l) < \epsilon$ for all $n, m, l \geq n_o$.

E. ***Definition :- [10]***

A dS_b –metric space $(\mathcal{A}, dS_b)$ is said to be dS_b– complete or complete dS_b– metric space if each dS_b-Cauchy sequence is dS_b-convergent in $\mathcal{A}$.

F. ***Definition :- [6]***

The self-mappings p *and* q defined on a metric space $(\mathcal{A}, \rho)$ are compatible if $\lim_{n\to\infty} \rho(pq\, x_n, qp\, x_n) = 0$ assuming that $\{x_n\}$ is a sequence in $\mathcal{A}$ with $\lim_{n\to\infty} px_n = \lim_{n\to\infty} qx_n = v$ for some v $\epsilon\ \mathcal{A}$.

G. ***Definition :-[5]***

The self-maps p *and* q of a set $\mathcal{X}$ are known to be weakly compatible if $pa = qa$ for some $a \in \mathcal{X}$ then $qpa = pqa$.

H. ***Definition:- [3]***

Let P and S be two self-maps of a metric space$(\mathcal{A}, \rho)$. The pair (P, S) is said to satisfy the property (E.A) if there is a sequence $\{x_n\}$ in $\mathcal{A}$ in order that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Sx_n = v$ for some $v \in \mathcal{A}$.

III. COMMON FIXED POINT THEOREM

A. ***Lemma:***

In a dS_b-metric space $(\mathcal{A}, dS_b)$, we have (i) $dS_b(a, a, d) \leq b\,dS_b(d, d, a)$ (ii) $dS_b(d, d, a) \leq b\,dS_b(a, a, d)$

Proof:

By rectangular inequality we have,

$$dS_b(a, a, d) \leq b[dS_b(a, a, a) + dS_b(a, a, a) + dS_b(d, d, a)]$$

$$= b[2dS_b(a, a, a) + dS_b(d, d, a)]$$

$= bdS_b(d,d,a)$

(i.e) $dS_b(a,a,d) \leq bdS_b(d,d,a)$

Similarly,

$$dS_b(d,d,a) \leq b[2dS_b(d,d,d) + dS_b(a,a,d)]$$
$$= bdS_b(a,a,d)$$

(i.e) $dS_b(d,d,a) \leq bdS_b(a,a,d)$

B. ***Lemma:***

In a d_qS_b-metric space $(\mathcal{B}, d_qS_b)$ we have (i) $d_qS_b(a,d,d) \leq bd_qS_b(d,d,a)$ (ii) $d_qS_b(d,a,a) \leq bd_qS_b(a,a,d)$

Proof:

By rectangular inequality we have,

$$d_qS_b(a,d,d) \leq b\left[d_qS_b(d,d,a) + d_qS_b(d,d,d) + d_qS_b(d,d,d)\right]$$
$$= bd_qS_b(d,d,a)$$

(i.e) $d_qS_b(a,d,d) \leq bd_qS_b(d,d,a)$

Similarly,

$$d_qS_b(d,a,a) \leq b\left[d_qS_b(a,a,d) + 2d_qS_b(a,a,a)\right]$$
$$= bd_qS_b(a,a,d)$$

(i.e) $d_qS_b(d,a,a) \leq bd_qS_b(a,a,d)$

C. ***Lemma:***

Let $(\mathcal{A}, dS_b)$ a dS_b-metric space with $b \geq 1$ and $\{x_n\}$ be a sequence dS_b- convergent to a. Then we have,

$$\frac{1}{2b} dS_b(a,d,d) \leq \lim_{n\to\infty} \inf dS_b(x_n,d,d) \leq \lim_{n\to\infty} sup\ dS_b(x_n,d,d) \leq 2bdS_b(a,d,d).$$

In particular, when $a = d$, $dS_b(x_n,d,d) \to 0\ as\ n \to \infty$.

Proof:

As in the definition we have,

$$dS_b(x_n,d,d) \leq b[dS_b(x_n,x_n,a) + dS_b(d,d,a) + dS_b(d,d,a)]$$
$$= bdS_b(x_n,x_n,a) + 2bdS_b(d,d,a) \qquad \text{........ (1)}$$
$$dS_b(a,d,d) \leq b[dS_b(a,a,x_n) + dS_b(d,d,x_n) + dS_b(d,d,x_n)]$$
$$= bdS_b(a,a,x_n) + 2bdS_b(d,d,x_n) \qquad \text{........ (2)}$$

Taking the upper limit as $n \to \infty$ in (1),

$$\lim_{n\to\infty} \sup dS_b(x_n,d,d) \leq b \lim_{n\to\infty} \sup dS_b(x_n,x_n,a) + 2b \lim_{n\to\infty} \sup dS_b(d,d,a)$$
$$\Longrightarrow \lim_{n\to\infty} \sup dS_b(x_n,d,d) \leq 2bdS_b(d,d,a)$$
$$= 2bdS_b(a,d,d)$$

Taking the lower limit as $n \to \infty$ in (2) we get,

$$\lim_{n\to\infty} \inf dS_b\,(a,d,d) \le b\,\lim_{n\to\infty} \inf dS_b(a,a,x_n) + 2b\,\lim_{n\to\infty} \inf dS_b(d,d,x_n)$$

$$dS_b(a,d,d) \le 2b\,\lim_{n\to\infty} \inf dS_b(d,d,x_n)$$

$$\lim_{n\to\infty} \inf dS_b(d,d,x_n) \ge \frac{1}{2b} dS_b(a,d,d)$$

Also we have,

$$\lim_{n\to\infty} \inf dS_b(d,d,x_n) \le \lim_{n\to\infty} \sup dS_b(d,d,x_n)$$

$$\therefore \frac{1}{2b} dS_b(a,d,d) \le \lim_{n\to\infty} \inf dS_b(x_n,d,d\,) \le \lim_{n\to\infty} \sup dS_b(x_n,d,d) \le 2b\, dS_b(a,d,d)$$

If $a = d$ then $dS_b(a,d,d) \to 0$ as $n \to \infty$.

$$\therefore \lim_{n\to\infty} dS_b(x_n,d,d) = 0.$$

D. ***Theorem :***

Let $(\mathcal{A}, dS_b)$ be a complete dS_b-metric space and p *and* s be two self-mappings on $(\mathcal{A}, dS_b)$ satisfies the following conditions:

(i) $p(\mathcal{A}) \subseteq s(\mathcal{A})$

(ii) p or s is dS_b- continuous.

(iii) $dS_b(pa,pd,pf) \le b\,[\alpha dS_b(pa,sd,sf) + \beta dS_b(sa,pd,sf) + \gamma dS_b(sa,sd,pf)]$ for every $a,d,f \in \mathcal{A}$ and $\alpha,\beta,\gamma \ge 0$ with $0 \le \alpha + 2\beta + 2\gamma < \frac{1}{b^2}$.

Then p *and* s have precisely one common fixed point in $\mathcal{A}$ assigned that p *and* s are compatible mappings.

Proof:

Let x_0 be a point in $\mathcal{A}$. Choosing a point $x_1 \in \mathcal{A}$ such that $px_0 = sx_1$, since by (i). In general, x_{n+1} can be taken as $t_n = px_n = sx_{n+1}$, n=0, 1, 2,……

Now from (iii) we have,

$$\begin{aligned} dS_b(px_n,px_{n+1},px_{n+1}) &\le b[\alpha dS_b(px_n,sx_{n+1},sx_{n+1}) + \beta dS_b(sx_n,px_{n+1},sx_{n+1}) + \gamma dS_b(sx_n,sx_{n+1},px_{n+1})] \\ &= b[\alpha dS_b(px_n,px_n,px_n) + \beta dS_b(px_{n-1},px_{n+1},px_n) + \gamma dS_b(px_{n-1},px_n,px_{n+1})] \\ &= b[\beta+\gamma] dS_b(px_{n-1},px_n,px_{n+1}) \end{aligned}$$

Now by rectangular inequality,

$$\begin{aligned} dS_b(px_{n-1},px_n,px_{n+1}) &\le b[dS_b(px_{n-1},px_{n-1},px_n) + dS_b(px_n,px_n,px_n) + dS_b(px_{n+1},px_{n+1},px_n) \\ &= b[dS_b(px_{n-1},px_{n-1},px_n) + dS_b(px_{n+1},px_{n+1},px_n)] \end{aligned}$$

$$\therefore\ dS_b(px_n,px_{n+1},px_{n+1}) \le b^2(\beta+\gamma)[dS_b(px_{n-1},px_{n-1},px_n) + dS_b(px_{n+1},px_{n+1},px_n)]$$

$$(1 - b^2\beta - b^2\gamma) dS_b(px_n,px_{n+1},px_{n+1}) \le b^2(\beta+\gamma) dS_b(px_{n-1},px_{n-1},px_n)$$

$$dS_b(px_n,px_{n+1},px_{n+1}) \le \frac{b^2(\beta+\gamma)}{(1 - b^2\beta - b^2\gamma)}\ dS_b(px_{n-1},px_{n-1},px_n)$$

$$= q dS_b(px_{n-1}, px_{n-1}, px_n)\ ,$$

where $q = \frac{b^2(\beta+\gamma)}{(1-b^2\beta-b^2\gamma)} < \frac{1}{b^2} < 1$

$dS_b(px_n, px_{n+1}, px_{n+1}) \leq q^n dS_b(px_0, px_0, px_1)$

$\therefore dS_b(t_n, t_m, t_m) \leq b[dS_b(t_n, t_n, t_{n-1}) + dS_b(t_{n+1}, t_{n+1}, t_n) + \cdots + dS_b(t_m, t_m, t_{m-1})],$

$n < m$

$$\leq b[q^n + q^{n+1} + \cdots + q^{m-1}]dS_b(t_0, t_0, t_1)$$

$$= bq^n[1 + q + q^2 + \cdots + q^{n-m-1}]dS_b(t_0, t_0, t_1)$$

$$= b\frac{q^n}{1-q}\ dS_b(t_0, t_0, t_1)$$

As $n, m \to \infty, dS_b(t_n, t_m, t_m) \to 0$.

Thus $\{t_n\}$ is dS_b-Cauchy in $\mathcal{A}$.

Since $\mathcal{A}$ is dS_b-complete, we have a point $a \in \mathcal{A}$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} px_n = \lim_{n\to\infty} sx_{n+1} = a$.

Now assuming that s is dS_b-continuous, we have $\lim_{n\to\infty} spx_n = \lim_{n\to\infty} ssx_{n+1} = sa$ and also p *and* s are compatible then,

$\lim_{n\to\infty} dS_b(psx_n\ , spx_n, spx_n) = 0$

$\Longrightarrow \lim_{n\to\infty}\ ps\,x_n = sa.$

From (iii),

$dS_b(psx_n, px_n, px_n)$

$$\leq b[\alpha dS_b(psx_n, sx_n, sx_n) + \beta dS_b(ssx_n, px_n, sx_n) + \ \gamma dS_b(ssx_n, sx_n, px_n)]$$

Letting $n \to \infty$ we get, $\lim_{n\to\infty} psx_n = \ \lim_{n\to\infty} px_n$

$\Longrightarrow sa = a$

Also from (iii),

$dS_b(px_n, pa, pa) \leq b[\alpha dS_b(px_n, sa, sa) + \beta dS_b(sx_n, pa, sa) + \gamma dS_b(sx_n, sa, pa)]$

Letting $n \to \infty$ we get, $\lim_{n\to\infty} fp = pa$

$\Longrightarrow a = pa$

$\therefore pa = sa = a.$

Thus a is a fixed point common to p *and* s.

Let a_1, a_2 be two common fixed points of p *and* s with $a_1 \neq a_2$.

We have $dS_b(a_2, a_1, a_1) > 0$ and

$dS_b(a_2, a_1, a_1) = dS_b(pa_2, pa_1, pa_1)$

$$\leq b[\alpha dS_b(pa_2, sa_1, sa_1) + \beta dS_b(sa_2, pa_1, sa_1) + \gamma dS_b(sa_2, sa_1, pa_1)]$$

$$= b[\alpha + \beta + \gamma]dS_b(a_2, a_1, a_1)$$

$\Longrightarrow dS_b(a_2, a_1, a_1) < dS_b(a_2, a_1, a_1)$

which is a contradiction.

Therefore $a_1 = a_2$ is the one and only common fixed point of $p\ and\ s$.

E. ***Corollary:***

Let $(\mathcal{A}, dS_b)$ be a dS_b - metric space and $p\ and\ s$ be two compatible mapping from $\mathcal{A}\ into\ itself$ satisfies

(i) $p(\mathcal{A}) \subseteq s(\mathcal{A})$
(ii) $p\ or\ s$ is dS_b continuous.
(iii) $dS_b(pa, pd, pf) \leq q dS_b(sa, sd, sf)$, for all $a, d, f \in \mathcal{A}, 0 < q < 1$.

Then $p\ and\ s$ have precisely one common fixed point in $\mathcal{A}$.

Proof:

Since $q < 1$, from the above Theorem D, $p\ or\ s$ have a common fixed point theorem in $\mathcal{A}$ which is a single point.

F. ***Theorem:***

Let $p\ and\ s$ be weakly compatible self-maps of a dS_b-metric space $(\mathcal{A}, dS_b)$ satisfying conditions (i) and (iii) of theorem D and either the subspaces $p(\mathcal{A})$ or $s(\mathcal{A})$ is dS_b –complete. Then $p\ and\ s$ have a common fixed point in $\mathcal{A}$.

Proof:

We have by Theorem D, $\{t_n\}$ is dS_b-Cauchy in $\mathcal{A}$.

Assume that $s(\mathcal{A})$ is a dS_b-complete subspace of $\mathcal{A}$, then the subsequence of $\{t_n\}$ must be dS_b-convergent in $s(\mathcal{A})$.

Let the $dS_b - limit\ point$ be $a \in \mathcal{A}$. Let $c = g^{-1}(a) \in \mathcal{A}$. Then $s(c) = a$.

Since $\{t_n\}$ contains a dS_b-convergent subsequence, we have $\{t_n\}$ is also a dS_b- convergent sequence.

Now we claim that, $f(c) = a$.

Put $a = c, d = x_n, f = x_n$ in condition (iii) of theorem D.

$$dS_b(pc, px_n, px_n) \leq b[\alpha dS_b(pc, sx_n, sx_n) + \beta dS_b(sc, px_n, px_n) + \gamma dS_b(su, sx_n, px_n)]$$

Letting $n \to \infty$ we get,

$$dS_b(pc, a, a) \leq b[\alpha dS_b(pc, a, a)]$$

$$\Longrightarrow pc = a.$$

$$\therefore pc = sc = a$$

That is c is a coincidence point of $p\ and\ s$.

Now by hypothesis $p\ and\ s$ are weakly compatible, it gives that $psc = spc \Longrightarrow pc = sc$.

Now we have to prove that $pa = a$

Suppose that $pa \neq a$ we get, $dS_b(pa, a, a) > 0$.

If $a = a, d = c, f = c$ in condition (iii) of theorem D then we get,

$$dS_b(pa, pc, pc) \leq b[\alpha dS_b(pa, sc, sc) + \beta dS_b(sa, pc, sc) + \gamma dS_b(sa, sc, pc)]$$

$$= b[\alpha dS_b(pa,a,a) + \beta dS_b(sa,a,a) + \gamma dS_b(sa,a,a)]$$

$$= b[\alpha + \beta + \gamma] dS_b(pa,a,a)$$

$dS_b(pa,a,a) \le dS_b(pa,a,a)$

which is a contradiction.

$\therefore fa = ga = a$

Thus 'a' is a common fixed point of p *and* s.

Now, assume that $a_1 \neq a_2$ be two common fixed points of p *and* s.

Then $pa_1 = sa_1 = a_1$ and $pa_2 = sa_2 = a_2$.

Now,

$$dS_b(a_1,a_2,a_2) = dS_b(pa_1,pa_2,pa_2)$$

$$\le b[\alpha dS_b(pa_1,sa_2,sa_2) + \beta dS_b(sa_1,pa_2,sa_2) + \gamma dS_b(sa_1,sa_2,pa_2)]$$

$$= b[\alpha + \beta + \gamma] dS_b(a_1,a_2,a_2)$$

$dS_b(a_1,a_2,a_2) \le dS_b(a_1,a_2,a_2)$

this is a contradiction.

Therefore $a_1 = a_2$ is a single common fixed point of p *and* s.

G. ***Example :***

Given that $\mathcal{A} = [0,1]$ and let dS_b be the dS_b-metric on $\mathcal{A}^3$ described as $dS_b(a,d,f) = [|d + f - 2a| + |f - d|]^2$, $a,d,f \in \mathcal{A}$. Then $(\mathcal{A}, dS_b)$ is a dS_b- metric space. Let $p,s: \mathcal{A}^3 \to \mathcal{A}^3$ be described as $pa = \frac{a}{6}$ and $sa = \frac{a}{2}$. Here we have p is dS_b -continuous and $p(\mathcal{A}) \subseteq s(\mathcal{A})$. Also we have $dS_b(pa,pd,pf) \le q\, dS_b(sa,sd,sf)$ is true for all $a,d,f \in \mathcal{A}$ with $a < d < f$ and $\frac{1}{18} \le q \le \frac{1}{b^2}$, b=2. Clearly 0 is the unique common fixed point.

IV. Property (E.A) In Dislocated S_B-metric space

A. ***Theorem:***

Let p *and* s be two self-mappings on dS_b-metric space $(\mathcal{A}, dS_b)$ satisfying condition (iii) in Theorem D and also the below conditions:

(i) The property (E.A) holds for p *and* s.

(ii) $s(\mathcal{A})$ is a dS_b-closed subspace in $\mathcal{A}$.

Then p *and* s have a common fixed point in $\mathcal{A}$ which is unique. Also given that p *and* s are weakly compatible self-mappings.

Proof:

Since by (i), there is a sequence $\{x_n\}$ in $\mathcal{A}$ with $\lim_{n\to\infty} px_n = \lim_{n\to\infty} sx_n = a \in \mathcal{A}$.

Also since $s(\mathcal{A})$is a dS_b- closed subspace of $\mathcal{A}$, we have every dS_b-convergent sequence of points of $s(\mathcal{A})$ has a dS_b-limit point in $s(\mathcal{A})$.

$\lim_{n\to\infty} px_n = a = \lim_{n\to\infty} sx_n = sc$, for some $c \in \mathcal{A}$

$\Longrightarrow a = sc \in s(\mathcal{A})$.

Now from (iii) of Theorem D we have,

$$dS_b(pc, px_n, px_n) \leq b[\alpha\, dS_b(pc, sx_n, sx_n) + \beta\, dS_b(sc, px_n, px_n) + \gamma\, dS_b(sc, sx_n, px_n)]$$

Letting $n \to \infty$ as the upper limit and by Lemma C we get,

$$dS_b(pc, a, a) \leq b[\alpha\, dS_b(pc, a, a) + \beta\, dS_b(sc, a, a) + \gamma\, dS_b(sc, a, a)]$$

$$= b\alpha\, dS_b(pc, a, a)$$

$$\leq (\alpha + 2\beta + 2\gamma) b\alpha\, dS_b(pc, a, a)$$

Since $0 \leq \alpha + 2\beta + 2\gamma < \frac{1}{b^2}$ and $b \geq 1$, we have $dS_b(pc, a, a) = 0 \Longrightarrow pc = a$.

∴ c is a coincidence point of p *and* s. (i.e) $pc = sc = a$

Since p *and* s are weakly compatible mappings, we get $pa = psc = spc = sa$.

Now we assert that 'a' is the common fixed point of p *and* s.

According to condition (iii) of Theorem D, we get

$$dS_b(pa, px_n, px_n) \leq b[\alpha dS_b(pa, sx_n, sx_n) + \beta dS_b(sa, px_n, sx_n) + \gamma dS_b(sa, sx_n, px_n)]$$

Taking the upper limit as $n \to \infty$ we get

$$dS_b(pa, a, a) \leq b[\alpha\, dS_b(pa, a, a) + \beta\, dS_b(sa, a, a) + \gamma\, dS_b\,(sa, a, a)]$$

$$= b[\alpha + \beta + \gamma] dS_b(pa, a, a\,)$$

Since $0 \leq \alpha + 2\beta + 2\gamma < \frac{1}{b^2}$ and $b \geq 1$ we have, $\lim_{n\to\infty} dS_b(pa, a, a) = 0 \Longrightarrow pa = a$ (i.e) $pa = a = sa$

Hence 'a' is the common fixed point of p *and* s.

As in theorem D, uniqueness follows.

B. ***Corollary:***

Let $(\mathcal{A}, dS_b)$ be a complete dS_b-metric space and p *and* s be two self-mappings on $(\mathcal{A}, dS_b)$ satisfying conditions (i) and (ii) of above theorem and $dS_b(pa, pd, pf) \leq q dS_b(sa, sd, sf)$ for every $a, d, f \in \mathcal{A}$ and $0 < q < 1$. Then p *and* s have exactly one common fixed point in $\mathcal{A}$ given that p *and* s are weakly compatible.

Proof:

Since $q < 1$, from the above theorem, p *and* s have exactly one common fixed point.

V. CONCLUSION

In summary, continuity and commutativity of the maps are minimized and the completeness of the space to the coincidence point is weakened. Also the property (E.A) obtains the act of containing range without continuity to the coincidence point.

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