Two Common coupled Fixed Point Theorems in S_b-Metric Spaces

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Abstract:

In this paper, we obtain twocommon coupled fixed point theorems in $\boldsymbol{S_b}$-metric spacesand obtain an example in support of our result.

Keywords: $\boldsymbol{S_b}$- Metric Space, S-metric spaces, Fixed point.

Classcification: 54H25, 47H10.

1. Introduction

In 1906, Maurice Frechet [1] introduced concept of metric spaces and many authors studied with reference to fixed point theorems. Fixed point theory is one of the most important topics in the study of non-linear analysis.

In 1931, W.A.Wilson[2] introduced the notion of quasi-metric spaces and proved some fixed point results in quasi-metric spaces.

In 1989, Backhtin[21] introduced the concept of b-metric space. In 1993, Czerwik [22] extended the results of b-metric spaces and many authors proved some fixed point results on b- metric spaces [21,22].

In 2012, S.Sedghi [3] introduced the concept of S- metric space which is a generalization if G-metric spaces and D^*-metric spaces and many authors proved some fixed point results on S- metric spaces [3,22,23,24,25].

In 2006, Gnana B.T.;Lakshmikantham[17] introduced the notion of coupled fixed point and proved some coupled fixed point results .

In 2018, Sastry[10] introduced the concept of quasi$S$-metric spaces and proved some fixed point results. Also Sastry[11] proved two fixed point theorems in quasi S-metric spaces.

In 2016, Sedghi[13] introduced the concept of S_b-metric spaces as a generalization of the b-metric spacesand recently Souayh and Mlaiki[12]introduced another concept of S_b- metric space and proved some fixed point results. Several researchers worked on S_b- metric spaces and proved some fixed point theorems on S_b- metric spaces [12,13,14,15,16,17].

The aim of this paper is to obtain two common coupled fixed point theorems in S_b-metric spaces as defined in [12] and obtain an example in support of our result. We obtain results of Bulbul. K. et al.[15] as corollaries.

2. Preliminaries

In this section, we give some known definitions and examples which we use in the next section.

Definition 2.1.[1]: A metric on a non empty set X is a function D : X × X → [0, ∞) such that for all$x, y, z \in X$thefollowingconditionshold:

(i) $D(x,x) = 0$ for all $x \in X$;

(ii) $D(x,y) = 0 = D(y,x) \Rightarrow x = y$for all $x, y \in X$;

(iii) $D(x,y) = D(y,x)$for all $x, y, \in X$;

(iv)$D(x,y) \leq D(x,z) + D(z,y)\ \ \forall x,y,z \in X.$

The pair (X, D) is called a metric space.

Definition 2.2.([2]): A quasi metric on a non empty set X is a function D: X × X → $[0, \infty)$ such that for all$x, y, z \in X$, the following conditions hold:

(i) $D(x,x) = 0$ for all $x \in X$;

(ii) $D(x,y) = 0 = D(y,x) \Leftrightarrow x = y$for all $x, y \in X$;

(iii)$D(x,y) \leq D(x,z) + D(z,y)\ \ \forall x,y,z \in X.$

Thepair(X, D)iscalleda quasi-metricspace.

Definition 2.3.([3]): Let X be a non empty set. An S-metric on X is a function $S: X \times X \times X \to [0, \infty)$ that satisfies the following conditions, for all $x, y, z, a \in X$;

(i) $S(x, y, z) = 0$ if and only if $x = y = z$;

(ii) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$.

The function S is called an S -metric on X and the pair (X, S) is called an S-metric space.

Example 2.4: Let $X = R^n, S(x, y, z) = |y - z - 2x| + |y - z|$, then (X, S) is an S-metric space.

Example 2.5: Let $X = R^n, S(x, y, z) = |x - z| + |y - z|$, then (X, S) is an S-metric space.

Example 2.6: Let (X, d) be a metric space. Define $S(x, y, z) = d(x, z) + d(y, z)$, for all $x, y, z \in X$, then S is an S-metric space.

Observation 2.7: Let (X, S) be an S-metric space. Then for all $x, y, z \in X$, we have

(i) $S(x, x, y) = S(y, y, x)$;

(ii) $S(x, x, z) \leq 2S(x, x, y) + S(z, z, y)$ and

(iii) $S(x, y, y) \leq S(x, x, y)$.

Definition 2.8([10]):A quasi S- metric space on a non empty set X is a function $q: X^3 \to [0, \infty)$ that satisfies the following conditions: for all $x, y, z \in X$

(i) $q(x, x, x) = q(y, y, y) = q(z, z, z) = q(x, y, z)$ if and only if $x = y = z$;

(ii) $\max\{q(x, x, t). q(y, y, t), q(z, z, t)\} \leq q(x, y, z)$ for $t \in \{x, y, z\}$;

(iii) $q(x, y, z)$ is invariant of any permutation of x, y, z;

(iv) $q(x, y, z) \leq q(x, z, t) + q(z, y, t) - q(z, z, t)$ for all $x, y, z, t \in X$.

The pair (X, q) is called a quasi S-metric space

Example 2.9: Let $X = \{[a, b]/a, b \in R^+, a \leq b\}$ and define $q([a, b], [c, d], [e, f]) = \max\{b, d, f\} - \min\{a, c, e\}$. Then $q: X^3 \to R^+$ is a quasi S-metric space.

Example 2.10: Let $X = \{0,1,2,3 \ldots.\}$ and $q: X^3 \to R^+$ be defined by $q(x, y, z) = \max\{x, y, z\}$ for all $x, y, z \in X$.Then (X, q) is a quasi S-metric space.

Example 2.11: Let $X = \{0,1,2,3 \ldots.\}$ and $q: X^3 \to R^+$ be defined by $q(x, y, z) = \max\{x^2, y^2, z^2\}$ for all $x, y, z \in X$.Then (X, q) is a quasi S-metric space.

Observation 2.12: $q(x, y, y) = q(x, x, y)$.

Now we give the definition of S_b-metric space given in [14].

Definition 2.13.([12]): Let X be a non empty set and $s \geq 1$ be a real number. Suppose S_b: X × X × X → $[0, \infty)$ is a function which satisfies the following conditions, for all $x, y, z, a \in X$

(i) $S_b(x, y, z) = 0$ if and only if $x = y = z$;

(ii) $S_b(x, x, y) = S_b(y, y, x)$ for all $x, y \in X$;

(iii) $S_b(x, y, z) \leq s(S_b(x, x, a) + S_b(y, y, a) + S_b(z, z, a))$ for all $x, y, z, a \in X$.

Then the function S_b is called an S_b -metric spaces with index s and the pair (X, S_b) is called S_b-metric space.

Remark 2.14: The class S_b-metric spaces is effectively larger than that of S-metric spaces.

Definition 2.15: Let (X, S_b) be an S_b- metric space and $\{x_n\}$ be a sequence in X, then the sequence $\{x_n\}$ is called convergent if there exist $z \in X$ such that $S_b(x_n, x_n, z) \rightarrow 0$ as $n \rightarrow \infty$. In this case we write $\lim_{n\to\infty} x_n = z$.

Definition 2.16: Let (X, S_b) be an S_b- metric space and $\{x_n\}$ be a sequence in X, then the sequence $\{x_n\}$ is called a Cauchy sequence if $S_b(x_n, x_n, x_m) \rightarrow 0$ as $n, m \rightarrow \infty$.

Definition 2.17: Let (X, S_b) be an S_b- metric space and $\{x_n\}$ sequence in X. Then (X, S_b) is said to be complete S_b-metric space if every Cauchy sequence $\{x_n\}$ in X converges to a point $x \in X$ so that $\lim_{n,m\to\infty} S_b(x_n, x_n, x_m) = \lim_{n,m\to\infty} S_b(x_n, x_n, x) = S_b(x, x, x)$.

Definition 2.18: An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping F: X × X → X if $F(x, y) = x$ and $F(y, x) = y$.

Definition 2.19: An element $(x, y) \in X \times X$ is called a coupled coincidence point of mapping F: X × X → X and g: X → X if $F(x, y) = gx$ and $F(y, x) = gy$, and (gx, gy) is called a coupled point of coincidence.

Definition 2.20: An element $(x, y) \in X \times X$ is called a common coupled fixed point of mapping F: X × X → X and g: X → X if $F(x, y) = gx = x$ and $F(y, x) = gy = y$.

Definition 2.21: Let X be a non-empty set and F: X × X → X and g: X → X. We say that F and g are commutative if $g(F(x, y)) = F(gx, gy)$ for all $x, y \in X$.

Definition 2.22: The mappings F: X × X → X and g: X → X are called ω-compatible if $g(F(x, y)) = F(gx, gy)$ whenever $F(x, y) = gx$ and $F(y, x) = gy$.

Lemma 2.23[15]: Let (X, S_b) be an S_b-metric space, and suppose that the sequence $\{x_n\}$ converges to x. Then $\frac{1}{s} S_b(y, y, x) \leq \lim_{n\to\infty} inf\, S_b(y, y, x_n) \leq \lim_{n\to\infty} sup\, S_b(y, y, x_n) \leq s S_b(y, y, x)$.

3. Main Results

Definition3.1: Let Ψ denote the class of all functions$\psi: [0, \infty) \rightarrow [0, \infty)$ such that ψ is increasing, continuous, $\psi(\mathrm{t}) < t$ for all $\mathrm{t} > 0$and $\psi(0) = 0$.

Theorem 3.2. Let (X, S_b)be anS_b-metric space with index s. Let$F: \mathrm{X} \times \mathrm{X} \rightarrow \mathrm{X}andg: X \rightarrow X$be two mappings such that,

$S_b(F(x, y), F(u, v), F(a, b)) \leq \frac{1}{s}\psi(\max\{S_b(gx, gu, ga), S_b(gy, gv, gb)\}$…………………(I)

for some $\psi \in \Psi$ and for all $\mathrm{x, y, u, v, a, b} \in \mathrm{X}$. Assume that ψ, F and g satisfy the following conditions:

(i) $F(X \times X) \subseteq g(X)$,

(ii) $g(X)$ is complete,

(iii) $F$and$g$ are ω-comptible,and

(iv) There is a $k > 1$ such that $2^m\psi^m(t) < kt$ for all $m$and $t > 0$.

(For example if we define $\psi(\mathrm{t}) = \frac{\mathrm{t}}{2}$ for $t \geq 0$ then $\psi \in \Psi$ and Ψ has property (iv))

Then F and $g$have a unique common fixed point, and this is of the form (x, x), that is there is a unique $x \in X$ such that $F(x, x) = gx = x$.

Proof: Let$x_0, y_0 \in X$

Since $F(X \times X) \subseteq g(X)$.

We can choose $x_1, y_1 \in X$ such that

$gx_1 = F(x_0, y_0)$and$gy_1 = F(y_0, x_0)$

Continuingthis process, we can construct two sequences$\{x_n\}, \{y_n\}$ in X

such that, $gx_{n+1} = F(x_n, y_n)$ and $gy_{n+1} = F(y_n, x_n)$ for all $n \geq 0$.

For $n \in N$, we have

$S_b(gx_n, gx_n, gx_{n+1}) \leq S_b((F(x_{n-1}, y_{n-1}), (F(x_{n-1}, y_{n-1}), (F(x_n, y_n))$

$$\leq \frac{1}{s}\psi(\max\{S_b(gx_{n-1}, gx_{n-1}, gx_n), S_b(gy_{n-1}, gy_{n-1}, gy_n)\} \ldots\ldots\ldots\ldots(3)$$

Similarly

$S_b(gy_n, gy_n, gy_{n+1}) \leq \frac{1}{s}\psi(\max\{S_b(gy_{n-1}, gy_{n-1}, gy_n), S_b(gx_{n-1}, gx_{n-1}, gx_n)\} \ldots\ldots\ldots\ldots(4)$

Write $P_n = \max\{ S_b(gx_n, gx_n, gx_{n+1}), S_b(gy_n, gy_n, gy_{n+1})\}$

Then from (3) and (4)

$$P_n \leq \frac{1}{s}\psi(\max\{S_b(gx_{n-1}, gx_{n-1}, gx_n), S_b(gy_{n-1}, gy_{n-1}, gy_n)\})$$

$$= \frac{1}{s}\psi(P_{n-1})$$

Therefore

$P_n \leq \frac{1}{s}\psi(P_{n-1}) \leq \frac{1}{s}\psi(\frac{1}{s}\psi(P_{n-2})) \leq \frac{1}{s}\psi(\psi(P_{n-2})) \leq \frac{1}{s}\psi^2(P_{n-2}) \leq \frac{1}{s}\psi^3(P_{n-3}) \leq \ldots\ldots\ldots\ldots \leq \frac{1}{s}\psi^n(P_0)$

For $m, n \in N$ with $m > n$

wehave

Then for $m \geq n$

Write $P_n^m = \max\{S_b(gx_n, gx_n, gx_m), S_b(gy_n, gy_n, gy_m)\}$

$P_n^m \leq \max\{ S_b(gx_n, gx_n, gx_{n+1}) + S_b(gx_n, gx_n, gx_{n+1}) + S_b(gx_m, gx_m, gx_{n+1}),$

$S_b(gy_n, gy_n, gy_{n+1}) + S_b(gy_n, gy_n, gy_{n+1}) + S_b(gy_m, gy_m, gy_{n+1})\}$

$\leq \max\{ 2S_b(gx_n, gx_n, gx_{n+1}) + S_b(gx_m, gx_m, gx_{n+1}), 2S_b(gy_n, gy_n, gy_{n+1}) + S_b(gy_m, gy_m, gy_{n+1})\}$

$\leq \max\{ 2\max\{ S_b(gx_n, gx_n, gx_{n+1}), S_b(gx_m, gx_m, gx_{n+1}), 2\max\{ S_b(gy_n, gy_n, gy_{n+1}), S_b(gy_m, gy_m, gy_{n+1})\}\}$

$= \max\{ 2\max\{ S_b(gx_n, gx_n, gx_{n+1}), S_b(gx_m, gx_m, gx_{n+1}), S_b(gy_n, gy_n, gy_{n+1}), S_b(gy_m, gy_m, gy_{n+1})\}\}$

$= \max\{ 2 \max\{ S_b(gx_n, gx_n, gx_{n+1}), S_b(gx_m, gx_m, gx_{n+1}), S_b(gy_n, gy_n, gy_{n+1}), S_b(gy_m, gy_m, gy_{n+1})\}\}$

$\leq 2 \max\{ S_b(gx_n, gx_n, gx_{n+1}), S_b(gx_{n+1}, gx_{n+1}, gx_m), S_b(gy_n, gy_n, gy_{n+1}), S_b(gy_{n+1}, gy_{n+1}, gy_m)\}$

$\leq 2 \max\{ \max\{ S_b(gx_n, gx_n, gx_{n+1}), S_b\left(gy_n, gy_n, gy_{n+1}\right)\}, \max \{S_b(gx_{n+1}, gx_{n+1}, gx_m), S_b(gy_{n+1}, gy_{n+1}, gy_m)\}\}$

$= 2 \max \{2 \max \left\{\frac{S_b(gx_n, gx_n, gx_{n+1})}{2}, \frac{S_b(gy_n, gy_n, gy_m)}{2}\right\}, \max \{S_b(gx_{n+1}, gx_{n+1}, gx_m), S_b(gy_{n+1}, gy_{n+1}, gy_m)\}\}$

$P_n^m \leq 2 \max\{ 2P_n, P_{n+1}^m\}$

$\leq 2 \max\{ 2P_n, 2 \max\{ 2P_{n+1}, P_{n+2}^m\}\}$

$\leq 2 \max\{ 2P_n, \max\{ 2^2 P_{n+1}, 2P_{n+2}^m\}\}$

$= 2 \max\{ 2P_n, 2^2 P_{n+1}, 2P_{n+2}^m\}\}$

$= 2^2 \max\{ P_n, 2P_{n+1}, P_{n+2}^m\}\}$

$\leq 2^2 \max\{ P_n, 2P_{n+1}, 2 \max\{ 2P_{n+2}, P_{n+3}^m\}\}$

$= 2^2 \max\{ P_n, 2P_{n+1}, \max\{ 2^2 P_{n+2}, 2P_{n+3}^m\}\}$

$= 2^2 \max\{ P_n, 2P_{n+1}, 2^2 P_{n+2}, 2P_{n+3}^m\}\}$

$\leq 2^2 \max\{ P_n, 2P_{n+1}, 2^2 P_{n+2}, 2^3 P_{n+3}, \ldots\ldots 2^{(m-n-1)} P_{n+(m-n-1)}, 2^{m-n} P_{n+(m-n)}^m\}$

$\leq 2^2 \max\{ P_n, 2P_{n+1}, 2^2 P_{n+2}, 2^3 P_{n+3}, \ldots\ldots 2^{(m-n-1)} P_{n+(m-n-1)}\}$

$\leq 2^2 \max\{\frac{1}{s}\psi^n(P_0), \frac{2}{s}\psi^{n+1}(P_0), \frac{2^2}{s}\psi^{n+2}(P_0), \ldots\ldots \frac{2^{(m-n-1)}}{s}\psi^{m-1}(P_0)\}$

$\leq \frac{1}{s}\psi^n(P_0) 2^2 \max\{ 1, 2\psi(P_0), 2^2\psi^2(P_0) \ldots\ldots, 2^{(m-n-1)}\psi^{(m-n-1)}(P_0)\}$

$\leq \frac{1}{s}\psi^n(P_0) 2^2 k P_0 \to 0 \text{as} n \to \infty$ (by condition(iv))

Therefore $\{gx_n\}, \{gy_n\}$are Cauchy sequencesin $g(X)$.

Since $g(X)$ is complete

we get $\{gx_n\}$ and $\{gy_n\}$are convergent to some gx and gy in $g(X)$ respectively.

Consider $S_b\big(gx_{n+1}, gx_{n+1}, F(x,y)\big) = S_b\big(F(x_n, y_n), F(x_n, y_n), F(x,y)\big)$

$\leq \frac{1}{s}\psi(\max\{S_b(gx_n, gx_n, gx), S_b(gy_n, gy_n, gy)\}) \to 0$ as $n \to \infty$

Therefore $gx_n \to F(x,y)$

Similarly $gy_n \to F(y,x)$,

But $gx_n \to gx$ and $gy_n \to gy$

Therefore $F(x,y) = gx$ and $F(y,x) = gy$

Thus (gx, gy) is coupled point of coincidence of F and g

Now we show that F and g have unique coupled point of coincidence.

Assume that (gx^*, gy^*) is also a coupled point of coincidence of F and g

i.e$F(x^*, y^*) = gx^*$ and $F(y^*, x^*) = gy^*$

Consider $S_b(gx, gx, gx^*) = S_b\big(F(x,y), F(x,y), F(x^*, y^*)\big)$

$\leq \frac{1}{s}\psi(\max\{S_b(gx, gx, gx^*), S_b(gy, gy, gy^*)\}$............(5)

Similarly $S_b(gy, gy, gy^*) \leq \frac{1}{s}\psi(\max\{S_b(gy, gy, gy^*), S_b(gx, gx, gx^*)\}$................(6)

From (5) and (6)

$\max\{S_b(gx, gx, gx^*), S_b(gy, gy, gy^*)\} \leq \frac{1}{s}\psi(\max\{S_b(gx, gx, gx^*), S_b(gy, gy, gy^*)\})$

Therefore $\max\{S_b(gx, gx, gx^*), S_b(gy, gy, gy^*)\} = 0$

Hence, $S_b(gx, gx, gx^*) = 0$, $S_b(gy, gy, gy^*) = 0$

Therefore $gx = gx^*$ and $gy = gy^*$

Thus F and g have a unique coupled point of coincidence.

Next we show that $gx = gy$

Consider $S_b(gx, gx, gy) = S_b\big(F(x, y), F(x, y), F(y, x)\big)$

$$\leq \frac{1}{s}\psi(\max\{S_b(gx, gx, gy), S_b(gy, gy, gx)\} \ldots\ldots(7)$$

Similarly $S_b(gy, gy, gx) \leq \frac{1}{s}\psi(\max\{S_b(gy, gy, gx), S_b(gx, gx, gy)\} \ldots\ldots(8)$

From (7) and (8)

$$\max\{S_b(gx, gx, gy), S_b(gy, gy, gx)\} \leq \frac{1}{s}\psi(\max\{S_b(gx, gx, gy), S_b(gy, gy, gx)\}$$

Therefore $S_b(gx, gx, gy) = 0$

Therefore $gx = gy$

Therefore $F(x, y) = gx = gy = F(y, x)$

As F and g are ω-compatible, by taking $u = gx$,

we get $gu = ggx = gF(x, y) = f(gx, gy) = F(u, u)$

This shows that (gu, gu) is a coupled point of coincidence of F and g

As coupled point of coincidence of F and g is unique.

But we have $gu = gx = u$

Hence $u = gu = F(u, u)$

i.eF and g have unique common coupled fixed point.

Corollary 3.3.[15] Let (X, S_b) be an S_b-metric space. $F: X \times X \rightarrow Xandg: X \rightarrow X$ be two mappings such that,

$S_b(F(x,y), F(u,v), F(a,b)) \leq \frac{1}{s}\psi(S_b(gx,gu,ga) + S_b(gy,gv,gb))$………………..…(II)

for some $\psi \in \Psi$ and for all $x,y,u,v,a,b \in X$. Assume that ψ, F and g satisfy the following conditions:

(i) $F(X \times X) \subseteq g(X)$,

(ii) $g(X)$ is complete, and

(iii) $F$and$g$ are ω-compatible.

Then F and $g$have a unique common fixed point, and it is of the form (x,x), that is, there is a unique $x \in X$ such that $F(x,x) = gx = x$.

Corollary 3.4. [15] Let (X, S_b) be an S_b-metric space. Let $F: X \times X \rightarrow Xandg: X \rightarrow X$ be two mappings such that, $S_b(F(x,y), F(u,v), F(a,b)) \leq \frac{k}{s}(S_b(gx,gu,ga) + S_b(gy,gv,gb))$

for all $x,y,u,v,a,b \in X$ as some $k \in (0,1)$.Assume that F and g satisfy the following conditions:

(i) $F(X \times X) \subseteq g(X)$,

(ii) $g(X)$ is complete, and

(iii) $F$and$g$ are ω-compatible.

If $k \in (0,\frac{1}{2})$, then F and $g$have a unique common fixed point, and it is of the form (x,x), that is, there is a unique $x \in X$ such that $F(x,x) = gx = x$.

Proof: The result follows from corollary 3.4 by taking $\psi(t) = kt$.

Corollary 3.5. [15] Let (X, S_b) be a complete S_b-metric space. Let$F: X \times X \rightarrow X$be a mapping such that, $S_b(F(x,y), F(u,v), F(a,b)) \leq \frac{1}{s}\psi(S_b(x,u,a) + S_b(y,v,b))$

for all $x,y,u,v,a,b \in X$. If $k \in (0,\frac{1}{2})$, then F has a unique coupled fixed point, and it is of the form (x,x), that is, there is a unique $x \in X$ such that $F(x,x) = x$.

Proof: The result follows from corollary 3.4 by taking $g = I$ (the identity mapping on X).

Corollary 3.6. [15] Let (X, S_b) be a complete S_b-metric space. Let$F$: X × X → Xbe a mapping such that, $S_b(F(x,y),F(u,v),F(a,b)) \leq \frac{k}{s}(S_b(x,u,a)+S_b(y,v,b))$

for all $x,y,u,v,a,b \in X$. If $k \in (0,\frac{1}{2})$, then F has a unique coupled fixed point, and it is of the form (x,x), that is, there is a unique $x \in X$ such that $F(x,x) = x$.

Proof: The result follows from corollary 3.4 by taking g = I (the identity mapping on X) and ψ(t) = kt.

Theorem 3.7. Let (X, S_b) be a complete S_b-metric space. Let$F$: X × X → Xand$g: X \to X$ be two mappings such that,

$S_b(F(x,y),F(u,v),F(a,b)) \leq \frac{1}{s^2}\psi(\max\{S_b(gx,gu,ga),S_b(gy,gv,gb)\}$ for some $\psi \in \Psi$. and for all $x,y,u,v,a,b \in X$. Assume that ψ, F and g satisfy the following conditions:

(i) $F(X \times X) \subseteq g(X)$,

(ii) $g(X)$ is complete,

(iii) $g$is continuous and commutes with F

(iv) There is a $k > 1$ such that $2^m\psi^m(t) < kt$ for all $m$and $t > 0$.

Then F and $g$have a unique common fixed point, and which is of the form (x,x), that is there is a unique $x \in X$ such that $F(x,x) = gx = x$.

Proof: Let $x_0, y_0 \in X$.

Since $F(X \times X) \subseteq g(X)$,

wecan choose $x_1, y_1 \in X$ such that

$gx_1 = F(x_0,y_0)$and$gy_1 = F(y_0,x_0)$

Continuing this process, we can construct two sequences$\{x_n\},\{y_n\}$ in X

such that, $gx_{n+1} = F(x_n,y_n)$ and $gy_{n+1} = F(y_n,x_n)$ for all $n \geq 0$.

For $n \in N$, we have

$S_b(gx_n,gx_n,gx_{n+1}) \leq S_b((F(x_{n-1},y_{n-1}),(F(x_{n-1},y_{n-1}),(F(x_n,y_n))$

$$\leq \frac{1}{s^2}\psi(\max\{S_b(gx_{n-1}, gx_{n-1}, gx_n), S_b(gy_{n-1}, gy_{n-1}, gy_n)\}\ldots\ldots\ldots\ldots(9)$$

Similarly

$S_b(gy_n, gy_n, gy_{n+1}) \leq$

$\frac{1}{s^2}\psi(\max\{S_b(gy_{n-1}, gy_{n-1}, gy_n), S_b(gx_{n-1}, gx_{n-1}, gx_n)\}\ldots\ldots\ldots\ldots(10)$

Write $Q_n = \max\{ S_b(gx_n, gx_n, gx_{n+1}), S_b(gy_n, gy_n, gy_{n+1})\}$

Then from (9)and (10)

$$Q_n \leq \frac{1}{s^2}\psi(\max\{S_b(gx_{n-1}, gx_{n-1}, gx_n), S_b(gy_{n-1}, gy_{n-1}, gy_n)\})$$

$$\leq \frac{1}{s^2}\psi(Q_{n-1})$$

Therefore $Q_n \leq \frac{1}{s^2}\psi(Q_{n-1}) \leq \frac{1}{s^2}\psi^2(Q_{n-2}) \leq \frac{1}{s^2}\psi^3(Q_{n-3}) \leq \ldots\ldots\ldots\ldots \leq \frac{1}{s^2}\psi^n(Q_0)$

For $m, n \in N$ with $m > n$

Write $Q_n^m = \max\{S_b(gx_n, gx_n, gx_m), S_b(gy_n, gy_n, gy_m)\}$

Then

$Q_n^m \leq \max\{ S_b(gx_n, gx_n, gx_{n+1}) + S_b(gx_n, gx_n, gx_{n+1}) + S_b(gx_m, gx_m, gx_{n+1}),$

$$S_b(gy_n, gy_n, gy_{n+1}) + S_b(gy_n, gy_n, gy_{n+1}) + S_b(gy_m, gy_m, gy_{n+1})\}$$

$\leq \max\{ 2S_b(gx_n, gx_n, gx_{n+1}) + S_b(gx_m, gx_m, gx_{n+1}), 2S_b(gy_n, gy_n, gy_{n+1}) + S_b(gy_m, gy_m, gy_{n+1})\}$

$\leq \max\{ 2\max\{ S_b(gx_n, gx_n, gx_{n+1}), S_b(gx_m, gx_m, gx_{n+1}), 2\max\{ S_b(gy_n, gy_n, gy_{n+1}), S_b(gy_m, gy_m, gy_{n+1})\}\}$

$= \max\{ 2\max\{ S_b(gx_n, gx_n, gx_{n+1}), S_b(gx_m, gx_m, gx_{n+1}), S_b(gy_n, gy_n, gy_{n+1}), S_b(gy_m, gy_m, gy_{n+1})\}\}$

$= \max\{ 2\max\{ S_b(gx_n, gx_n, gx_{n+1}), S_b(gx_m, gx_m, gx_{n+1}), S_b(gy_n, gy_n, gy_{n+1}), S_b(gy_m, gy_m, gy_{n+1})\}\}$

$$\leq 2\max\{S_b(gx_n, gx_n, gx_{n+1}), S_b(gx_{n+1}, gx_{n+1}, gx_m), S_b(gy_n, gy_n, gy_{n+1}), S_b(gy_{n+1}, gy_{n+1}, gy_m)\}$$

$$\leq 2\max\{\max\{S_b(gx_n, gx_n, gx_{n+1}), S_b(gy_n, gy_n, gy_{n+1})\}, \max\{S_b(gx_{n+1}, gx_{n+1}, gx_m), S_b(gy_{n+1}, gy_{n+1}, gy_m)\}\}$$

$$= 2\max\{2\max\left\{\frac{S_b(gx_n, gx_n, gx_{n+1})}{2}, \frac{S_b(gy_n, gy_n, gy_m)}{2}\right\}, \max\{S_b(gx_{n+1}, gx_{n+1}, gx_m), S_b(gy_{n+1}, gy_{n+1}, gy_m)\}\}$$

$$Q_n^m \leq 2\max\{2Q_n, Q_{n+1}^m\}$$

$$\leq 2\max\{2Q_n, 2\max\{2Q_{n+1}, Q_{n+2}^m\}\}$$

$$\leq 2\max\{2Q_n, \max\{2^2Q_{n+1}, 2Q_{n+2}^m\}\}$$

$$= 2\max\{2Q_n, 2^2Q_{n+1}, 2Q_{n+2}^m\}\}$$

$$= 2^2\max\{Q_n, 2Q_{n+1}, Q_{n+2}^m\}\}$$

$$\leq 2^2\max\{Q_n, 2Q_{n+1}, 2\max\{2Q_{n+2}, Q_{n+3}^m\}\}$$

$$= 2^2\max\{Q_n, 2Q_{n+1}, \max\{2^2Q_{n+2}, 2Q_{n+3}^m\}\}$$

$$= 2^2\max\{Q_n, 2Q_{n+1}, 2^2Q_{n+2}, 2Q_{n+3}^m\}\}$$

$$\leq 2^2\max\{Q_n, 2Q_{n+1}, 2^2Q_{n+2}, 2^3Q_{n+3}, \dots\dots, 2^{(m-n-1)}Q_{n+(m-n-1)}, 2^{m-n}Q_{n+(m-n)}^m\}$$

$$\leq 2^2\max\{Q_n, 2Q_{n+1}, 2^2Q_{n+2}, 2^3Q_{n+3}, \dots\dots, 2^{(m-n-1)}Q_{n+(m-n-1)}\}$$

$$\leq 2^2\max\{\frac{1}{s^2}\psi^n(Q_0), \frac{2}{s^2}\psi^{n+1}(Q_0), \frac{2^2}{s^2}\psi^{n+2}(Q_0), \dots\dots, \frac{2^{(m-n-1)}}{s^2}\psi^{m-1}(Q_0)\}$$

$$\leq \frac{1}{s^2}\psi^n(Q_0)2^2\max\{1, 2\psi(Q_0), 2^2\psi^n(Q_0)\dots\dots, 2^{(m-n-1)}\psi^{(m-n-1)}(Q_0)\}$$

$$\leq \frac{1}{s^2}\psi^n(Q_0)2^2kQ_0 \to 0 \text{ as } n \to \infty$$

Therefore $\{gx_n\}, \{gy_n\}$are Cauchy sequence in $g(X)$

Since $g(X)$ is complete

we get $\{gx_n\}$ and $\{gy_n\}$ are convergent to some x and y in $g(X)$respectively.

Since g is continuous, $\{ggx_n\}$ and $\{ggy_n\}$ are convergent to some gx and gy respectively.

Also, since g and F commute,

we have $ggx_{n+1} = gF(x_n, y_n) = F(gx_n, gy_n)$ and $ggy_{n+1} = gF(y_n, x_n) = F(gy_n, gx_n)$.

Thus $S_b(F(x,y), F(x,y), ggx_{n+1}) = S_b(F(x,y), F(x,y), F(gx_n, gy_n))$

$$\leq \frac{1}{s^2}\psi(\max\{S_b(gx, gx, gx_n), S_b(gy, gy, gy_n)\})$$

Applying the lemma 2.23, then we get

$$\frac{1}{s}S_b(F(x,y), F(x,y), gx) \leq \lim_{n\to\infty} sup\, S_b(F(x,y), F(x,y), F(gx_n, gy_n))$$

$$\leq \lim_{n\to\infty} sup \frac{1}{s^2}\psi(\max\{S_b(gx, gx, gx_n), S_b(gy, gy, gy_n)\})$$

$$\leq \frac{1}{s^2}\psi(\max\{S_b(gx, gx, gx), S_b(gy, gy, gy)\})$$

$$= 0$$

Therefore $S_b(F(x,y), F(x,y), gx) = 0$

Hence $gx = F(x,y)$.

Similarly, we get $gy = F(y,x)$.

Thus (x,y) is a coupled coincidence point of F and g

Now $S_b(gx, gx, gy) = S_b\big(F(x,y), F(x,y), F(y,x)\big)$

$$\leq \frac{1}{s^2}\psi(\max\{S_b(gx,gx,gy),S_b(gy,gy,gx)\})$$

$$= \frac{1}{s^2}\psi(\max\{S_b(gx,gx,gy),S_b(gx,gx,gy)\})$$

$$\leq \frac{1}{s^2}\psi S_b(gx,gx,gy)$$

Therefore $S_b(gx,gx,gy) = 0$

Similarly $S_b(gy,gy,gx) = 0$

This implies that$gx = gy$.

Thus we obtain $F(x,y) = gx = gy = F(y,x)$.

Using the lemma 2.23, we get $\frac{1}{s}S_b(gx,gx,x) \leq \lim_{n\to\infty} sup\, S_b(gx,gx,gx_{n+1})$

$$\leq \lim_{n\to\infty} sup\, S_b\big(F(x,y),F(x,y),F(x_n,y_n)\big)$$

$$\leq \lim_{n\to\infty} sup \frac{1}{s^2}\psi(\max\{S_b(gx,gx,gx_n),S_b(gy,gy,gy_n)\})$$

$$\leq \frac{1}{s^2}\psi(\max\{S_b(gx,gx,x),S_b(gy,gy,y)\})$$

Hence, $S_b(gx,gx,x) \leq \frac{1}{s}\psi(\max\{S_b(gx,gx,x),S_b(gy,gy,y)\})$……….(11)

Similarly $S_b(gy,gy,y) \leq \frac{1}{s}\psi(\max\{S_b(gy,gy,y),S_b(gx,gx,x)\})$……….(12)

From (11) and (12)

$$\max\{S_b(gx,gx,x),S_b(gy,gy,y)\} \leq \frac{1}{s}\psi(\max\{S_b(gx,gx,x),S_b(gy,gy,y)\})$$

Hence, $\max\{S_b(gx,gx,x),S_b(gy,gy,y)\} = 0$

Therefore $S_b(gx, gx, x) = 0, S_b(gy, gy, y) = 0.$

Therefore $gx = x$ and $gy = y$

Hence we have $gx = x = F(x, x)$

Let $x^* \in X$ with $x^* \neq x$ such that $x^* = gx^* = F(x^*, x^*)$

Consider $S_b(x, x, x^*) = S_b(F(x, y), F(x, y), F(x^*, x^*)$

$$\leq \frac{1}{s^2}\psi(\max\{S_b(gx, gx, gx^*), S_b(gx, gx, gx^*)\})$$

$$\leq \frac{1}{s^2}\psi S_b(gx, gx, gx^*)$$

$$\leq \frac{1}{s^2}\psi S_b(x, x, x^*)$$

Therefore$S_b(x, x, x^*) \leq \frac{1}{s^2}\psi S_b(x, x, x^*)$

Therefore $S_b(x, x, x^*) = 0$

Thus we obtain $x = x^*$

Hence F and g have a unique common coupled fixed point.

Corollarly3.8[15]:Let (X, S_b) be a complete S_b-metric space. Let$F$: X × X → X and $g: X \to X$ be two mappings such that,

$S_b(F(x, y), F(u, v), F(a, b)) \leq \frac{1}{s^2}\psi(S_b(gx, gu, ga) + S_b(gy, gv, gb))$ for some $\psi \in \Psi$. and for all $x, y, u, v, a, b \in X$. Assume that F and g satisfy the following conditions:

(i) $F(X \times X) \subseteq g(X)$,

(ii) $g(X)$ is complete,

(iii) g is continuous and commutes with F

Then F and $g$have a unique common fixed point, and which is of the form (x, x), that is, there is a unique $x \in X$ such that $F(x, x) = gx = x$.

Corollarly 3.9[15]: Let(X, S_b) be a completeS_b-metric space. Let$F$: X × X → Xand$g: X \to X$ be two mappings such that,

$S_b(F(x, y), F(u, v), F(a, b)) \leq \frac{k}{s^2}(S_b(gx, gu, ga) + S_b(gy, gv, gb))$

for all $x, y, u, v, a, b \in X$. Assume that F and g satisfy the following conditions:

(i) $F(X \times X) \subseteq g(X)$,

(ii) $g(X)$ is complete, and

(iii) g is continuous and commutes with F

If $k \in (0, \frac{1}{2})$, then F and $g$have a unique common fixed point, and which is of the form (x, x), that is, there is a unique $x \in X$ such that $F(x, x) = gx = x$.

Proof: The result follows from corollary 3.8 by taking $\psi(t) = kt$.

Corollarly 3.10[15]:Let(X, S_b) be a complete S_b-metric space. Let$F: X \times X \rightarrow X$be a mapping such that, $S_b(F(x,y), F(u,v), F(a,b)) \leq \frac{1}{s^2}\psi(S_b(x,u,a) + S_b(y,v,b))$

for all $x, y, u, v, a, b \in X$. If $k \in (0, \frac{1}{2})$, then F has a unique coupled fixed point, and which is of the form (x, x), that is, there is a unique $x \in X$ such that $F(x, x) = x$.

Proof: The result follows from corollary 3.8 by taking $g = I$ (the identity mapping on X)

Corollary 3.11. [15] Let (X, S_b) be a complete S_b-metric space. Let$F: X \times X \rightarrow X$be a mapping such that, $S_b(F(x,y), F(u,v), F(a,b)) \leq \frac{k}{s^2}(S_b(x,u,a) + S_b(y,v,b))$

for all $x, y, u, v, a, b \in X$. If $k \in (0, \frac{1}{2})$, then F has a unique coupled fixed point, and which is of the form (x, x), that is, there is a unique $x \in X$ such that $F(x, x) = x$.

Proof: The result follows from corollary 3.8 by taking $g = I$ (the identity mapping on X) and $\psi(t) = kt$.

Example 3.12: Let $X = [0, \infty)$.Define$S_b: X^3 \rightarrow R^+$ by $S_b(x,y,z) = \max\{|x-z|, |y-z|\}$ for all $x, y, z \in X$. Thus (X, S_b) is an S_b-metric space with $s = 1$.

Define$F(x,y) = \frac{1}{128}(x+y), g(x) = \frac{x}{4}$and $\psi(t) = \frac{t}{8}$, next we show that F and g are ω-compatible.

$$\text{In fact} \begin{cases} F(x,y) = gx \\ F(y,x) = gy \end{cases} \Leftrightarrow x = y = 0$$

Thus $(g0, g0)$ is the uniquecoupled point of coincidence of the mapping F and g.Obviously

$$F(g0, g0) = g\big(F(0,0)\big) = 0$$

Therefore F and g are ω-compatible.

Clearly (X, S_b) is a S_b-metric space.

Now $S_b(F(x,y), F(u,v), F(a,b)) \leq \frac{1}{s}\psi(\max\{S_b(gx,gu,ga), S_b(gy,gv,gb)\}$

$$\leq \frac{1}{1}\psi(\max\{\max\{S_b\left(\frac{x}{4},\frac{u}{4},\frac{a}{4}\right), S_b\left(\frac{y}{4},\frac{v}{4},\frac{b}{4}\right)\}\})$$

$$\leq \psi(\max\{\max\left\{\left|\frac{x}{4}-\frac{a}{4}\right|,\left|\frac{u}{4}-\frac{a}{4}\right|,\left|\frac{y}{4}-\frac{b}{4}\right|,\left|\frac{v}{4}-\frac{b}{4}\right|\right\}\})$$

$$\leq \psi(\max\left\{\left|\frac{x}{4}-\frac{a}{4}\right|,\left|\frac{u}{4}-\frac{a}{4}\right|,\left|\frac{y}{4}-\frac{b}{4}\right|,\left|\frac{v}{4}-\frac{b}{4}\right|\right\})$$

$$\leq \frac{1}{32}\max\{|x-a|,|u-a|,|y-b|,|v-b|\}=\text{R.H.S}$$

Now $S_b\big(F(x,y), F(u,v), F(a,b)\big) = S_b(\frac{1}{128}(x+y), \frac{1}{128}(u+v), \frac{1}{128}(a+b))$

$$= \frac{1}{128}S_b(x+y, u+v, a+b)$$

$$= \frac{1}{128}\max\{|x+y-a-b|, |u+v-a-b|\}$$

$$\leq \frac{1}{128}\max\{|x-a|+|y-b|, |u-a|+|v-b|\}$$

$$\leq \frac{1}{128}\max\{2\max\{|x-a|,|y-b|\}, 2\max\{|u-a|,|v-b|\}\}$$

$$\leq \frac{2}{128}\max\{|x-a|,|y-b|,|u-a|,|v-b|\}$$

$$\leq \frac{1}{64}\max\{|x-a|,|y-b|,|u-a|,|v-b|\}=\text{L.H.S}$$

Therefore $L.H.S \leq R.H.S$

Hence F and g satisfy all the conditions of theorem 3.1

Hence F and g have unique common coupled fixed point.

Infact $F(0,0) = g(0)$.

Example 3.13: Let $X = [0,\infty)$. Define $S_b: X^3 \to R^+$ by $S_b(x,y,z) = \max\{|x-z|, |y-z|\}$ for all $x,y,z \in X$. Thus (X, S_b) is a complete S_b-metric space with $s = 1$.

Define $F(x,y) = \frac{1}{64}(x+y)$, $g(x) = \frac{x}{4}$ and $\psi(t) = \frac{t}{3}$

Now $S_b(F(x,y), F(u,v), F(a,b)) \leq \frac{1}{s^2}\psi(\max\{S_b(gx,gu,ga), S_b(gy,gv,gb)\}$

$$\leq \frac{1}{1}\psi(\max\{\max\{S_b\left(\frac{x}{4},\frac{u}{4},\frac{a}{4}\right), S_b\left(\frac{y}{4},\frac{v}{4},\frac{b}{4}\right)\}\})$$

$$\leq \psi(\max\{\max\left\{\left|\frac{x}{4}-\frac{a}{4}\right|,\left|\frac{u}{4}-\frac{a}{4}\right|,\left|\frac{y}{4}-\frac{b}{4}\right|,\left|\frac{v}{4}-\frac{b}{4}\right|\right\}\})$$

$$\leq \psi(\max\left\{\left|\frac{x}{4}-\frac{a}{4}\right|,\left|\frac{u}{4}-\frac{a}{4}\right|,\left|\frac{y}{4}-\frac{b}{4}\right|,\left|\frac{v}{4}-\frac{b}{4}\right|\right\})$$

$$\leq \frac{1}{12}\max\{|x-a|,|u-a|,|y-b|,|v-b|\}=\text{R.H.S}$$

Also$S_b\big(F(x,y),F(u,v),F(a,b)\big) = S_b(\frac{1}{64}(x+y),\frac{1}{64}(u+v),\frac{1}{64}(a+b))$

$$=\frac{1}{64}S_b(x+y,u+v,a+b)$$

$$=\frac{1}{64}\max\{\,|x+y-a-b|,|u+v-a-b|\}$$

$$\leq\frac{1}{64}\max\{\,|x-a|+|y-b|,|u-a|+|v-b|\}$$

$$\leq\frac{1}{64}\max\{\,2\max\{\,|x-a|,|y-b|\},2\max\{|u-a|,|v-b|\}\}$$

$$\leq\frac{2}{64}\max\{\,|x-a|,|y-b|,|u-a|,|v-b|\}$$

$$\leq\frac{1}{32}\max\{\,|x-a|,|y-b|,|u-a|,|v-b|\}=\text{L.H.S}$$

Therefore $L.H.S \leq R.H.S$

Hence F and g satisfy all the conditions of theorem 3.7

Hence F and g have unique common coupled fixed point.

Infect $F(0,0) = g(0) = 0$.

Conflict of interest

The authors declare that there is no conflict of interest.

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