STAR COLORING OF MIDDLE GRAPH OF CERTAIN SNAKE GRAPHS

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Abstract : A proper vertex coloring of a graph is a star coloring if every path of G on four vertices is not 2-colored. The star chromatic number of G is the minimum number of colors needed to star color G and it is denoted by $\chi_S(G)$. In this paper star coloring of certain snake graphs are studied. Some structural properties of them are discussed and their star chromatic number were obtained.

Keywords :Double triangular snake graph, Diamond snake Graph coloring, Middle graph, Star chromatic number, Star coloring

Classification number: 05C15, 05C76

1. Introduction

Let G be a finite, undirected graph with no loops and multiple edges. The graph G has the vertex set V (G) and the edge set E (G). Graph coloring is coloring of G such that no two adjacent vertices share the same color.

A proper vertex coloring of a graph is a **star coloring** if every path of G on four vertices is not 2-colored. The **star chromatic number** of G is the minimum number of colors needed to star color G and it is denoted by $\chi_S(G)$.

The middle graph of G denoted by M(G), is defined as follows :

The vertex set of M(G) is V(G)∪E(G) in which two elements are adjacent in M(G) if the following conditions hold.

(i) x,y ∈ E(G) and x, y are adjacent in G.(ii) x ∈V(G), y ∈ E(G) and y is incident on x in G.

A double triangular snake graph is obtained from a path $v_1, v_2, v_3, \ldots\ldots v_{n+1}$ by joining v_i and v_{i+1} to a new vertex w_i to another new vertex u_i .It has 3n+1 vertices and 5n edges.

A diamond snake graph is obtained from a path $v_1, v_2, v_3, \ldots\ldots v_{n+1}$ by joining v_i and v_{i+1} to a new vertex w_i to another new vertex u_i .It has 3n+1 vertices and 5n edges.

2. Structural properties of middle graph of Double Triangular and Diamond snake graphs

- Number of vertices in $M[D(T_n)]$,p=8n+1
- Number of vertices in $M[D_n]$,p=7n+1
- Maximum degree in $M[D(T_n)]$, $\Delta=7$
- Minimum degree in $M[D(T_n)]$, $\delta = 3$
- Maximum degree in $M[D_n]$, $\Delta=6$
- Minimum degree in $M[D_n]$, $\delta = 2$

3. Star coloring of M [D(T_n)] and M [D_n]

Theorem 3.1 : If n≥2,then the star chromatic number of middle graph of double triangular snake graph is 3n+1. $(i.e)$ $\chi_S [M(D(T_n))] = 3n + 1$, n ≥2

Proof:

Let D (T_n) be the double triangular snake graph with 3n+1 vertices and 5n edges

Let $\{v_1, v_2 \ldots\ldots v_{n+1}$, $u_1, u_2 \ldots\ldots u_n$, $w_1, w_2 \ldots\ldots w_n\}$ be the vertices of the double triangular snake graph D(T_n)

Assume that each edge (v_i, v_{i+1}), (u_i, v_j) and (v_j, w_i) , i=1,2,3,….n and j=1,2,3…n+1 is sub-divided by the vertices x_i, e_{ij} and f_{ij} for , i=1,2,3,….n and j=1,2,3…n+1 respectively.

Now the coloring assignment of M (D (T_n) is as follows,

Color the vertices $v_1, v_2 \ldots\ldots v_{n+1}$, $u_1, u_2 \ldots\ldots u_n$ and $w_1, w_2 \ldots\ldots w_n$ with c_1.

Color the sub-divided vertices $e_{11}, e_{12} \ldots\ldots e_{j,j+1}$ with c_2 , $c_3 \ldots\ldots c_{2n+1}$

Color the vertices $f_{11}, f_{12}\ldots\ldots f_{j,j+1}$ with $c_{2n+1}, c_{2n}, \ldots\ldots c_2$

Atlast ,color the sub-divided vertices $x_1, x_2\ldots\ldots x_n$ with $c_{2n+2}, c_{2n+3}\ldots\ldots c_{3n+1}$.

$C(v_i) = 1$ for $1 \leq i \leq n+1$, $c(u_i) = 1$ for $1 \leq i \leq n$, $c(w_i) = 1$ for $1 \leq i \leq n$,

An easy check shows that no path on 4 vertices is bicolored which satisfies the definition of star coloring. Hence the minimum number of colors needed to star coloring for $M[(D(T_n))]$ is $3n+1$.

$(i.e)$ $\chi_S [M(T_n)] = 3n+1$

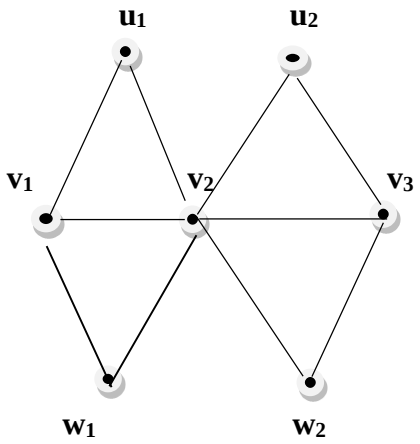

Figure 1: Double Triangular snake graph $D(T_2)$

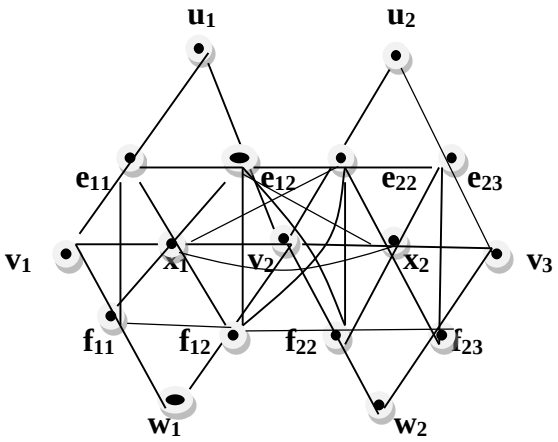

Figure 2: Middle graph of Double Triangular snake graph

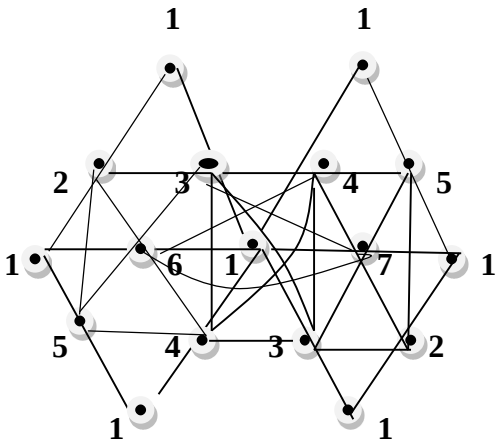

Figure 3 : Middle graph of Double Triangular snake graph D(T_2)

Theorem 3.2:

For the middle graph of diamond snake graph ,the star chromatic number is 2n+1

$(i.e)$ $\chi_S[M(D_n)] = 2n+1$, $n \geq 2$

Proof:

Let D_n be the diamond snake graph 3n+1 vertices and 4n edges

Let $\{v_1, v_2 \ldots\ldots v_{n+1}, u_1, u_2 \ldots\ldots u_n, w_1, w_2, \ldots w_n\}$ be the vertices of the diamond snake graph D_n.

Now by definition of middle graph, each edge of graph is sub-divided by a new vertex.

Now the coloring assignment M (D_n) is as follows

Color the vertices $\{v_1, v_2 \ldots\ldots\ldots v_{n+1}, u_1, u_2 \ldots\ldots\ldots u_n, w_1, w_2, \ldots w_n\}$ with c_1.

Color alternatively the sub-divided vertices $e_{11}, e_{12} \ldots\ldots e_{j,j+1}$ with $c_2, c_3 \ldots c_{2n+1}$

Color alternatively the sub-divided vertices $f_{11}, f_{12} \ldots\ldots f_{j,j+1}$ with $c_{2n+1}, c_{2n}, \ldots\ldots c_2$

$c(v_i) = 1$ for $1 \leq i \leq n+1$,

$c(u_i) = 1$ for $1 \leq i \leq n$,

$c(w_i) = 1$ for $1 \leq i \leq n$,

Hence we observe that no path on four vertices is bicolored.

This is a proper star coloring.

Hence the star chromatic number of M (D_n) *is* 2n+1.

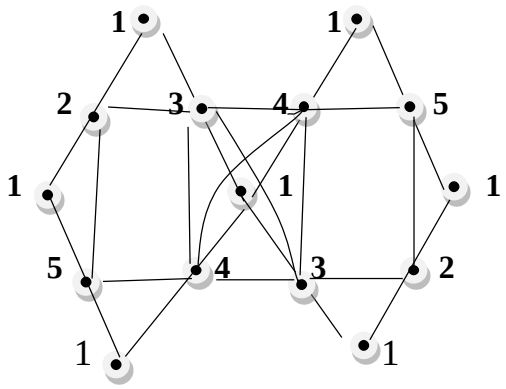

Figure 4: Middle graph of Diamond snake graph

Conclusion

In this paper, we have investigated the star chromatic number of middle graph of double triangular snake graph and diamond graph and established some theorems. This work can be further extended for various graphs.

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