Derivations of KM-Algebra

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Abstract: In this paper, a new notion,left derivation,right derivation and derivation of KM-algebra are introduced,some useful examples are discussed and related properties are investigated. Definition of left derivation,right derivation and derivation of KM-algebra along with various propositions are provided and exhibited with their respective proofs.

IndexTerms – **left derivation, right derivation and derivation of KM-algebra.**

1. INTRODUCTION

Algebras of logic form important class of algebras,among many algebraic structures. Some of these examples are BCK-algebras [8], BCI-algebras [9], BCH-algebras [7], KU-algebras [24], SU-algebras [12] and others. The above are strongly connected with logic. For instance, BCI-algebras introduced by Iseki [9] in 1966 have connections with BCI-logic.The two classes of logical algebras are BCK and BCI-algebras which were introduced by Imai and Iseki
[9, 10] in 1966 and have been vastly investigated by many researchers. It is known that
the class of BCK-algebras is a proper subclass of the class of BCI-algebras.
The notion of left-right (right-left) f-derivations of BCI-algebras was introduced by Zhan and Liu in 2005.Investigation of some fundamental properties was done by Abujabal and Al-shehri [1] in the year 2006 and proved some results on derivations of BCI-algebras. In 2007, Abujabal and Al-shehri [2] introduced the new concept of left derivations of BCI-algebras. In 2009, Javed and Aslam [11] investigated some results of f-derivations and some fundamental properties of BCI-algebras.

Nisar[22] introduced the very interesting concept of right F-derivations and left F-derivations of BCI-algebras. Prabpayak and Leerawat [24] investigated the notions of left-right (right-left) derivations of BCC-algebras. In 2012, Al-shehri and Bawazeer[4] studied the notion of left-right (right-left) t-derivations of BCC-algebras. Muhiuddin and Al-roqi [17] investigated the notion of (regular) (α; β)-derivations of BCI-algebras. In 2013, Bawazeer, Al-shehri andBabusal [6] studied the notion of generalized derivations of BCC-algebras. Lee [14] introduced a new kind of derivations of BCI-algebras. In 2014, Al-roqi [3] introduced the notion of generalized (regular) (α; β)-derivations of BCI-algebras. Muhiuddin and Al-roqi [19] introduced the notion of generalized left derivations of BCI-algebras. Ardekani and Davvaz [5] introduced the notion of (f; g)-derivations of BCI-algebras. In 2016, Sawika, Intasan, Kaewwasri and Iampan [25] introduced the notions of (l; r)-derivations, (r; l)-derivations and derivations of UP-algebras and investigated some related properties.In this paper, we introduce the notion of (l,r)derivations,(r,l) derivations and derivations of KM-algebras which is the generalization of the notion of derivations [25], some useful examples are discussed, and related properties are investigated.
Before we begin our study, we will introduce to the definition of a KM-algebra.

2. Preliminaries

In this section we site the fundamental definitions that will be used in the sequel

Definition 2.1

A *BCK-algebra* is an algebra(X,*,0)of type(2,0) satisfying the following conditions:

I) $(x*y)*(x*z)\leq(z*y)$

ii) $x*(x*y)\leq y$

iii) $x\leq x$

iv) $x\leq y$ and $y\leq x \Rightarrow x=y$

v) $0\leq x\rightarrow x=0$, where $x\leq y$ is defined by $x*y=0$, for all $x,y,z\epsilon X$.

Definition 2.2

A *BCI-algebra* is an algebra(X,*,0)of type(2,0) satisfying the following conditions:

I) (x*y)*(x*z)≤(z*y)

ii) x*(x*y)≤y

iii)x≤x

iv)x≤y and y≤x ⇒x=y

v)x≤0⇒x=0,where x≤y is defined by x*y=0,for all x,y,zϵX.

Definition 2.3

A *Q-algebra* is an algebra(X,*,0)of type(2,0) satisfying the following conditions:

I) x*x=0

ii) x*0=x

iii)(x*y)*z=(x*z)*y, where x≤y is defined by x*y=0,for all x,y,zϵX.

Definition 2.4

A *d-algebra* is an algebra(X,*,0)of type(2,0) satisfying the following conditions:

I) x*x=0

ii) 0*x=0

iii)x*y=0 and y*x=0 imply x=y, for all x, y ϵ X.

Definition 2.5

A *PS-algebra* is an algebra(X,*,0)of type(2,0) satisfying the following conditions:

I) x*x=0

ii) x*0=0

iii)x*y=0 and y*x=0 imply x=y,for all x,yϵX.

Remark:

- Every BCK-algebra is a BCI-algebra but not the converse.
- Every BCI-algebra is a BCH-algebra but not the converse.
- Every BCH-algebra is a Q-algebra but not the converse.
- Every BCK-algebra is a d-algebra but not the converse.

3. KM-Algebras and its Properties

Definition 3.1

A ***KM-algebra*** is an algebra(X,*,0)of type(2,0) satisfying the following conditions:

I) x*x=0

ii) x*0=0

iii)(x*y)*z=(x*z)*y

iii)x*y=0 and y*x=0 imply x=y, for all x, y, z ϵ X.

Proposition 3.2

In any KM-algebra (X,*,0) with x≤y,the following holds good for all x,yϵX.

i) x*(y*x) =y*(x*x)

ii) y*(x*(y*x))=0

iii) (x*(x*y))*y=(x*x)*(y*y)

iv) y*(y*(y*x))=y*x

v) x*(x*y)=x*y

vi) y*(x*(x*y))=0

vii) (x*y)*0=(x*0)*(y*0)

viii) (x*y)*x=0*y.

4. Derivations of KM-Algebras

For a KM-algebra, we denote x^y= y*(y*x), $\forall x, y \in x$.

Definition 4.1

A self-map d:A$\rightarrow$A is called an (l,r) derivation of A if it satisfies the identity d(x.y)^(x.d(y)) for all x,y$\in$A.

If A satisfies the identity d(x.y)=(x.d(y))^(d(x).y), $\forall x, y \in A$, then d is called the right-left derivation of A.

If d is both (l,r)&(r,l) derivation, then we say that d is a derivation of KM-Algebra.

Example 4.2

1) Let A={0, a_1, a_2, a_3} be a KM-algebra.

*	0	a_1	a_2	a_3
0	0	a_1	a_2	a_3
a_1	0	0	a_3	a_2
a_2	0	a_3	0	a_1
a_3	0	a_1	a_2	0

Define a map d: A$\rightarrow$A by

$$d(x)=\begin{cases} a3 & \text{if } x=0 \\ a3 & \text{if } x=a1 \\ a3 & \text{if } x=a2 \\ 0 & \text{if } x=a3 \end{cases}$$

d(0* a_1) = (d(0)* a_1)^(0*d(a_1))

= (a_3* a_1)^(0* a_2)

= (a_1)^ a_2

= a_2*(a_2* a_1)

= a_2*(a_3)

= a_1

d(0* a_1) = (0*d(a_1))^ (d(0)* a_1)

= (0* a_2)^(a_3* a_1)

$= (a_2)\wedge a_1$

$= a_1 * (a_1 * a_2)$

$= a_1 * (a_3)$

$= a_2$

$\therefore$ d is both left (l,r) & (r,l)derivation. Hence d is a derivation of X.

Example 4.3

Let Z be the set of all integers "-" the minus operation on Z.Then (Z,-,0) is a KM-algebra.Let d(x)=x-1, $\forall x \in Z$ .Then $(d(x)-y)\wedge(x-d(y)) = (x-1-y)\wedge(x-(y-1))$

$= (x-y-1)\wedge(x-y+1)$

$= (x-y+1)\wedge(x-y+1-x+y+1)$

$= x-y+1-2$

$= x-y-1$

$= d(x-y)$

$\Rightarrow$d is a (l,r)-derivation of X.

In particular,Now d(1-0) $= ((d(1)-0)\wedge(1-d(0))$

$= ((0-0)\wedge(1-(-1))$

$= 0\wedge 2$

=2-(2-0)

d(1)=0

Therefore d is a (l,r)-derivation of X.

But, d(1-0) $= ((1-d(0))\wedge(d(1)-0))$

$= ((1-(-1))\wedge(0-0))$

$= 2\wedge 0$

=0-(0-2)

d(1)=2≠0

Therefore d is not a (r,l)-derivation of X.

Definition 4.4

A self map d of a KM-algebra X is said to be regular if d(0)=0.

Proposition 4.5

Let d be a (l,r)-derrivation of KM-algebra X. Then

(i) d(0) = d(x)*x, $\forall x \in X$.

(ii) d is 1-1.

(iii)If d is regular, then it is the identity map.

(iv)If there is an element x϶X, such that d(x)=x,then d is the identity map.

(v) If there is an element x϶X such that d(y)*x=0 or x*d(y)=0 $\forall y \in X$, then d(y)=x, $\forall y \in X$ that is d is a constant.

Proof:

(i) Let $x \in X$ .Then $x * x = 0\,(by\,definition)$
d(0) =d(x*x)
=(d(x)*x)∧(x*d(x))
=(x*d(x))*[(x*d(x))*(d(x)*x)]
=d(x)*x (by property v).

(ii) Let $x, y \in X$, such that d(x)=d(y)
Then by (i) d(0)=d(x)*x
Also d(0)= d(y)*y
$\Rightarrow$ d(x)*x=d(y)*y
$\Rightarrow$ d(x)*x=d(x)*y
$\Rightarrow$ x=y.(by left cancellation law in KM-algebra)

(iii) Suppose that d is regular and $x \in X$,then d(0)=0
$\Rightarrow$ d(0)=d(x)*x
$\Rightarrow$ 0=d(x)*x
$\Rightarrow$ d(x)*d(x)=d(x)*x
$\Rightarrow$ d(x)=x,for all $x \in X$,(by left cancellation law)
$\Rightarrow$ d is the identity map

(iv) Suppose d(x)=x for some $x \in X$ .Then d(x)*x=d(0)
$\Rightarrow$ d is the identity map.

(v)follows directly.

Proposition 4.6

Let d be (r,l)-derivation of KM-algebra X.Then

(i) d(0)=x*d(x), $\forall x \in X$

(ii) d(x)=d(x)∧x, $\forall x \in X$

(iii) d is 1-1

(iv) If d is regular,then it is the identity map.

(v) If there is an element $x \in X$,such that d(x)=x,then d is the identity map.

(vi) If there is an element $x \in X$ such that d(y)*x=0 or x*d(y)=0, $\forall y \in X$,then d(y)=x, $\forall y \in X$,that is d is constant.

Proof:-

(i) Let $x \in X$ .Then x*x=0.

d(0) = d(x*x)

=(x*d(x))∧(d(x)*x)

=(d(x)*x)*[(d(x)*x)*(x*d(x)]

=x*d(x)(by property v)

(ii) Let $x \in X$.

d(x)*0=d(x)*[d(x)*(x*(x*d(x)))]

=d(x)*[d(x)*(x*d(0))]

$\Rightarrow$ d(x)*(x*d(0))=0

$\Rightarrow$ d(x)*(x*d(0))=d(x)*d(x)

$\Rightarrow$ d(x)=x*d(0) (by left cancellation law)

$\Rightarrow$ d(x)=x*(x*d(x))

d(x)=d(x)∧x, $\forall x \in X$.

(iii) Let $x, y \in X$,such that d(x)=d(y)

d(0)=x*d(x)

d(0)=y*d(y)

x*d(x)=y*d(y)

$\Rightarrow$ x*d(x)=y*d(x)

$\Rightarrow$ x=y

Therefore d is 1-1.

(iv) Let d be regular and $x \in X$.

$\Rightarrow$ d(0)=0

$\Rightarrow$ 0=x*d(x)

$\Rightarrow$ d(x)*d(x)=x*d(x)

$\Rightarrow$ d(x)=x, $\forall x \in X$

$\Rightarrow$ d is the identity map.

(v) Suppose d(x)=x, for some $x \in X$.

Then x*d(x)=0

$\Rightarrow$ d(0)=0

$\Rightarrow$ d is the identity map.

(vi) Follows directly

Proposition 4.7

Let A be a KM-algebra.Then every (r,l)-derivation,d:A→A with derivation is regular.

Proof:-

Given d:A→A is (r,l)-derivation.

d(0) = d(x*x)

=(x*d(x))∧(d(x)*x)

=0∧(d(x)*x)

=(d(x)*x)*[(d(x)*x)*0]

=[d(x)*x]*o

d(0)=0.

Therefore d is regular.

Lemma 4.8

If d:A→A is a (l,r)-derivation of KM-algebra A.Then d is regular.

Proof:-

Given d:A→A is a (l,r)-derivation.

d(0) =d(a*a)

=(d(a)*a)∧(a*d(a))

=0∧(a*d(a))

=(a*d(a))*[(a*d(a))*0]

=(a*d(a))*0

d(0)=0.

Therefore d is regular.

Theorem 4.9

Let A be a KM-algebra, $\exists$ a*0=a, $\forall a \in A$.

(i) If d is a (l,r)-derivation of A, then d(a)=d(a)∧a, $\forall a \in A$.

(ii) If d is a (r,l)-derivation of A, then d(a)=a∧d(a), $\forall a \in A$.

Proof:-

Let A be a KM-algebra.

(i) d(a)=d(a*0)

=(d(a)*0)∧(a*d(0))

=(d(a)*0)∧(a*0)

=(d(a)∧a), $\forall a \in A$.

(ii) d(a)=d(a*0)

=(a*d(0))∧(d(a)*0)

=(a*0)∧d(a)

d(a)=a∧d(a), $\forall a \in A$.

Lemma 4.10

Let d be a self –map of KM-algebra A. Then (a*(a*d(a)))*a=(d(a)*(d(a)*a))*a.

Proof:-

d(a)=d(a)∧a

=a*(a*d(a))

d(a)*a=(a*(a*d(a)))*a ______________(1)

d(a)=a∧d(a)

d(a)=d(a)*(d(a)*a)

d(a)*a=(d(a)*(d(a)*a))*a_____________(2)

from (1) & (2)

(a*(a*d(a)))*a=(d(a)*(d(a)*a))*a

Lemma 4.11

If A be a KM-algebra and let d be a derivation .Then the condition hold $\forall a \in A$.

(i) d(a*d(a))=0

(ii) d(d(a)*a)=0

Proof:-

(i) Let d be a (l,r)-derivation of A.

d(a*d(a))=(d(a)*d(a))∧(a*d(d(a)))

=0 ∧ (a*d(d(a)))

=(a*d(d(a)))*[a*d(d(a))*0]

=0

(ii) Let d be a (r,l)-derivation of A.

d(d(a)*a)=(d(a)*d(a))∧(d(d(a))*a)

=0 ∧ (d(d(a))*a)

=(d(d(a))*a)*[(d(d(a))*a)*0]

=0.

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