BIPOLAR INTERVAL VALUED MULTI FUZZY GENERALIZED SEMIPRE CONNECTED SPACES

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ABSTRACT: In this paper, bipolar interval valued multi fuzzy generalized semipre connected spaces is defined and introduced. Using this definitions, some theorems are introduced.

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KEYWORDS: Bipolar interval valued multi fuzzy subset, bipolar interval valued multi fuzzy topological space, bipolar interval valued multi fuzzy interior, bipolar interval valued multi fuzzy closure, bipolar interval valued multi fuzzy continuous mapping, bipolar interval valued multi fuzzy generalized semi-preclosed set, bipolar interval valued multi fuzzy generalized semi-preopen set, bipolar interval valued multi fuzzy generalized semi-preclosed mapping, bipolar interval valued multi fuzzy generalized semi-preopen mapping.

INTRODUCTION: The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [19] in the year 1965, the subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper C.L.Chang [4] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces many researchers like, and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] has introduced semipreclosed sets and Dontchev [5] has introduced generalized semipreclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by saraf and khanna [15]. Tapas kumar mondal and S.K.Samantha [12] have introduced the topology of interval valued fuzzy sets. Jeyabalan.R and Arjunan [8, 9] have introduced interval valued fuzzy generalized semi pre continuous mapping. After that interval valued fuzzy generalized semi pre continuous mapping has been generalized into interval valued intuitionistic fuzzy generalized semi pre continuous mapping by S.Vinoth and K.Arjunan[17, 18]. The interval valued fuzzy set has been extended into the bipolar interval valued multi fuzzy topological spaces. R.Selvam et.al [16] have defined and introduced the bipolar interval valued multi fuzzy generalized semipreclosed sets. In this paper, we introduce bipolar interval valued multi fuzzy generalized semi-preconnected spaces and some properties are investigated.

1. PRELIMINARIES:

Definition 1.1[19]. Let X be a non-empty set. A **fuzzy subset A** of X is a function A: X → [0, 1].

Definition 1.2[14]. A **multi fuzzy subset** A of a set X is defined as an object of the form A = { ⟨ x, A_1(x), A_2(x), A_3(x), …, A_n(x) ⟩ / x∈X}, where A_i : X→[0, 1] for all i and i = 1, 2, …, n.

Definition 1.3[19]. Let X be any nonempty set. A mapping A : X→D[0, 1] is called an interval valued fuzzy subset (briefly, IVFS) of X, where $D[0,1]$ denotes the family of all closed subintervals of [0, 1].

Definition 1.4[19]. A interval valued multi fuzzy subset A of a set X with degree n is defined as an object of the form A = { ⟨ x, A_1(x), A_2(x), A_3(x), …, A_n(x) ⟩ / x∈X}, where A_i : X→D[0, 1] for all i and i = 1, 2, …, n.

Definition 1.5[10]. A bipolar valued fuzzy set A in X is defined as an object of the form A = {⟨ x, M(x), N(x) ⟩/ x∈X}, where M : X→ [0, 1] and N : X→ [−1, 0]. The positive membership degree M(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree N(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.6. A = { ⟨ a, 0.7, −0.5 ⟩, ⟨ b, 0.3, −0.8 ⟩, ⟨ c, 0.2, −0.4 ⟩ } is a bipolar valued fuzzy subset of X = { a, b, c }.

Definition 1.7[10]. A bipolar valued multi fuzzy set A in X with degree n is defined as an object of the form A = {⟨x, M_1(x), M_2(x), M_3(x), …, M_n(x), N_1(x), N_2(x), N_3(x), …, N_n(x) ⟩ / x∈X}, where M_i : X→ [0, 1] and N_i : X→ [−1, 0] for all i and i = 1, 2, …, n.. The positive membership degrees M_i(x) denotes the satisfaction degrees of an element x to the property corresponding to a bipolar valued multi fuzzy set A and the negative membership degrees N_i(x) denotes the satisfaction degrees of an element x to some implicit counter-property corresponding to a bipolar valued multi fuzzy set A.

Example 1.8. A = { ⟨ a, 0.5, 0.4, 0.7, −0.2, −0.5, −0.8 ⟩, ⟨ b, 0.3, 0.7, 0.3, −0.3, −0.4, −0.6 ⟩, ⟨ c, 0.5, 0.8, 0.4, −0.5, −0.2, −0.9 ⟩ } is a bipolar valued multi fuzzy subset of X = { a, b, c } with degree 3.

Definition 1.9[16]. A bipolar interval valued fuzzy set A in X is defined as an object of the form A = {⟨ x, M(x), N(x) ⟩/ x∈X}, where M : X→D[0, 1] and N : X→D[−1, 0]. The positive membership interval degree M(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar interval valued fuzzy set A and the negative membership interval degree N(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar interval valued fuzzy set A.

Example 1.10. A = { ⟨ a, [0.3, 0.9], [−0.5, −0.4] ⟩, ⟨ b, [0.2,0.9], [−0.9, −0.5] ⟩, ⟨ c, [0.5, 0.8], [−0.8, −0.6] ⟩} is a bipolar interval valued fuzzy subset of X ={ a, b, c }.

Definition 1.11[16]. A bipolar interval valued multi fuzzy set A in X with degree n is defined as an object of the form A = {⟨ x, $M_1(x)$, $M_2(x)$, $M_3(x)$, …, $M_n(x)$, $N_1(x)$, $N_2(x)$, $N_3(x)$, …, $N_n(x)$ ⟩ / x∈X}, where $M_i : X \to D[0, 1]$ and $N_i : X \to D[-1, 0]$ for all i and i = 1, 2, …, n.. The positive membership degrees $M_i(x)$ denotes the satisfaction degrees of an element x to the property corresponding to a bipolar interval valued multi fuzzy set A and the negative membership degrees $N_i(x)$ denotes the satisfaction degrees of an element x to some implicit counter-property corresponding to a bipolar interval valued multi fuzzy set A.

Example 1.12. A = { ⟨ a, [0.3, 0.7], [0.2, 0.6], [0.5, 0.9], [−0.3, −0.2], [−0.6, −0.3], [−0.8, −0.3] ⟩, ⟨ b, [0.6, 0.9], [0.1, 0.9], [0.5, 0.5], [−0.3, −0.2], [−0.5, −0.3], [−0.6, −0.4] ⟩, ⟨ c, [0.5, 0.8], [0.3, 0.6], [0.4, 0.9], [−0.5, −0.2], [−0.8, −0.5], [−0.9, −0.7] ⟩ } is a bipolar interval valued multi fuzzy subset of X = { a, b, c } with degree 3.

Definition 1.13[16]. Let A = ⟨ M_i, N_i ⟩ and B = ⟨ O_i, P_i ⟩ be any two bipolar interval valued multi fuzzy subsets of a set X with degree n. We define the following relations and operations:
(i) $A \subseteq B$ if and only if $M_i(x) \le O_i(x)$ and $N_i(x) \ge P_i(x)$ for all x in X and for all i.
(ii)A =B if and only if $M_i(x) = O_i(x)$ and $N_i(x) = P_i(x)$ for all x in X and for all i.
(iii) $(A)^c$ = { ⟨ x, $(M_i)^c(x)$, $(N_i)^c(x)$ ⟩ / x∈X }.
(iv) $A \cap B$ = { ⟨ x, rmin{ $M_i(x)$, $O_i(x)$ }, rmax{ $N_i(x)$, $P_i(x)$ } ⟩ / x∈X }.
(v) $A \cup B$ = { ⟨ x, rmax { $M_i(x)$, $O_i(x)$}, rmin{ $N_i(x)$, $P_i(x)$ }⟩ / x∈X }.

Remark 1.14. $\bar{0}$ = { ⟨ x, [0, 0], [0, 0], …[0, 0] ⟩ : x∈X} and $\bar{1}$ = { ⟨ x, [1, 1], [1, 1],..,[1, 1], [−1, −1], [−1, −1],…, [−1, −1] ⟩ : $x \in X$ }.

Definition 1.15[16]. Let X be a set and $\Im$ be a family of bipolar interval valued multi fuzzy subsets of X. The family $\Im$ is called a bipolar interval valued multi fuzzy topology (BIVMFT) on X if $\Im$ satisfies the following axioms

(i) $\bar{0}, \bar{1} \in \Im$ (ii) If { A_i ; i∈I } $\subseteq \Im$, then $\bigcup_{i=1}^{\infty} A_i \in \Im$

(iii) If $A_1, A_2, A_3, \ldots A_n \in \Im$, then $\bigcap_{i=1}^{n} A_i \in \Im$.

The pair (X, $\Im$) is called a bipolar interval valued multi fuzzy topological space (BIVMFTS). The members of $\Im$ are called bipolar interval valued multi fuzzy open sets (BIVMFOS) in X. An bipolar interval valued multi fuzzy subset A in X is said to be bipolar interval valued multi fuzzy closed set (BIVMFCS) in X if and only if $(A)^c$ is a BIVMFOS in X.

Definition 1.16[16]. Let (X, $\Im$) be a BIVMFTS and A be a BIVMFS in X. Then the bipolar interval valued multi fuzzy interior and bipolar interval valued multi fuzzy closure are defined by $bivmfint(A) = \cup \{G : G$ is a BIVMFOS in X and $G \subseteq A\}$, $bivmfcl(A) = \cap$ { K: K is a BIVMFCS in X and A $\subseteq K$}. For any BIVMFS A in (X, $\Im$), we have $bivmfcl(A^c) = (bivmfint(A))^c$ and $bivmfint(A^c) = (bivmfcl(A))^c$.

Definition 1.17[16]. A BIVMFS A of a BIVMFTS (X, $\Im$) is said to be a
(i) bipolar interval valued multi fuzzy regular closed set (BIVMFRCS for short) if
$A = bivmfcl\big(bivmfint(A)\big)$

(ii) bipolar interval valued multi fuzzy semiclosed set (BIVMFSCS for short) if $bivmfint(bivmfcl(A)) \subseteq A$

(iii) bipolar interval valued multi fuzzy preclosed set (BIVMFPCS for short) if $bivmfcl(bivmfint(A)) \subseteq A$

(iv) bipolar interval valued multi fuzzy α closed set (BIVMF**α**CS for short) if $bivmfcl\left(bivmfint(bivmfcl(A))\right) \subseteq A$

(v) bipolar interval valued multi fuzzy β closed set (BIVMF**β**CS for short) if $bivmfint\left(bivmfcl(bivmfint(A))\right) \subseteq A$.

Definition 1.18[16]. A BIVMFS A of a BIVMFTS (X, ℑ) is said to be a
(i) bipolar interval valued multi fuzzy generalized closed set (BIVMFGCS for short) if bivmfcl(A) ⊆ U, whenever A⊆ U and U is a BIVMFOS
(ii) bipolar interval valued multi fuzzy regular generalized closed set (BIVMFRGCS for short) if biv$mfcl(A) \subseteq U$, whenever A⊆ U and U is a BIVMFROS.

Definition 1.19[16]. A BIVMFS A of a BIVMFTS (X, ℑ) is said to be a
(i) bipolar interval valued multi fuzzy semipreclosed set (BIVMFSPCS for short) if there exists a BIVMFPCS B such that biv$mfint(B) \subseteq A \subseteq B$
(ii) bipolar interval valued multi fuzzy semipreopen set (BIVMFSPOS for short) if there exists a BIVMFPOS B such that B $\subseteq A \subseteq bivmfcl(B)$.

Definition 1.20[16]. Two BIVMFSs A and B are said to be not q-coincident if and only if A ⊆ B^c.

Definition 1.21[16]. Let A be a BIVMFS in a BIVMFTS (X, ℑ). Then the bipolar interval valued multi fuzzy semipre interior of A ($bivmfspint(A)$ for short) and the bipolar interval valued multi fuzzy semipre closure of A (biv$mfspcl(A)$ for short) are defined by biv$mfspint(A)$= ∪ {$G : G$ is a BIVMFSPOS in X and $G \subseteq A$}, biv$mfspcl(A)$ =∩ { $K : K$ is a BIVMFSPCS in X and A ⊆ K }. For any BIVMFS A in $(X,\Im)$, we have bivmf$spcl$(A^c) = (bivmf$spint(A)$)^c and bivmf$spint$(A^c) = (bivmf$pcl(A)$)^c.

Definition 1.22[16]. A BIVMFS A in BIVMFTS (X, ℑ) is said to be a bipolar interval valued multi fuzzy generalized semipreclosed set (BIVMFGSPCS for short) if biv$mfspcl(A) \subseteq U$ whenever A ⊆ U and U is a BIVMFOS in (X,ℑ).

Example 1.23. Let $X = \{a, b\}$ and $\Im = \{\overline{0}, G, \overline{1}\}$ is a BIVMFT on X, where G = { ⟨ a, [0.5, 0.5], [0.6, 0.6], [0.4, 0.4], [−0.4, −0.4], [−0.5, −0.5], [−0.3, −0.3] ⟩, ⟨ b, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [−0.3, −0.3], [−0.4, −0.4], [−0.2, −0.2] ⟩}. And the BIVMFS A = { ⟨ a, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [−0.3, −0.3], [−0.4, −0.4], [−0.2, −0.2] ⟩, ⟨ b, [0.2, 0.2], [0.3, 0.3], [0.1, 0.1], [−0.1, −0.1], [−0.2, −0.2], [−0.05, −0.05] ⟩ } is a BIVMFGSPCS in (X,ℑ).

Definition 1.24[16]. The complement A^c of a BIVMFGSPCS A in a BIVMFTS (X, ℑ) is called a bipolar interval valued multi fuzzy generalized semi-preopen set (BIVMFGSPOS) in X.

Definition 1.25[16]. A BIVMFTS (X, $\Im$) is called a bipolar interval valued multi fuzzy semi-pre $T_{1/2}$ space (BIVMFSPT$_{1/2}$), if every BIVMFGSPCS is a BIVMFSPCS in X .

Definition 1.26. Let (X, $\Im$) and (Y, σ) be BIVMFTSs. Then a map h: X→Y is called a (i) bipolar interval valued multi fuzzy continuous (BIVMF continuous) mapping if $h^{-1}(B)$ is BIVMFOS in X for all BIVMFOS B in Y.
(ii) a bipolar interval valued multi fuzzy closed mapping (BIVMFC mapping) if h(A) is a BIVMFCS in Y for each BIVMFCS A in X.
(iii) bipolar interval valued multi fuzzy semi-closed mapping (BIVMFSC mapping) if $h(A)$ is a BIVMFSCS in Y for each BIVMFCS A in X.
(iv) bipolar interval valued multi fuzzy preclosed mapping (BIVMFPC mapping) if $h(A)$ is a BIVMFPCS in Y for each BIVMFCS A in X.
(v) bipolar interval valued multi fuzzy semi-open mapping (BIVMFSO mapping) if $h(A)$ is a BIVMFSOS in Y for each BIVMFOS A in X.
(vi) bipolar interval valued multi fuzzy generalized semi-preopen mapping (BIVMFGSPO mapping) if $h(A)$ is a BIVMFGSPOS in Y for each BIVMFOS A in X.
(vii) bipolar interval valued multi fuzzy generalized semi-preclosed mapping (BIVMFGSPC mapping) if $h(A)$ is a BIVMFGSPCS in Y for each BIVMFCS A in X.

Theorem 1.27. For any BIVMFS A in (X, τ) where X is a BIVMFSPT$_{1/2}$ space, A∈BIVMFGSPO(X) if and only if for every BIVMFP $p_{(\alpha, \beta)}$∈A, there exists a BIVMFGSPOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

2. SOME PROPERTIES.

Definition 2.1. An BIVMFT (X, τ) is said to be an bipolar interval valued multi fuzzy generalized semipre connected space (BIVMFGSP connected space for short) if the only BIVMFS which are both an BIVMFGSPOS and an BIVMFGSPCS are 0_X and 1_X.

Theorem 2.2. Every BIVMFGSP connected space is an BIVMFC$_5$- connected space but not conversely.

Proof. Let (X, τ) be an BIVMFGSP connected space. Suppose (X, τ) is not an BIVMFC$_5$-connected space, then there exists a proper BIVMFS A which is both an BIVMFOS and an BIVMFCS in (X, τ). That is, A is both an BIVMFGSPOS and an BIVMFGSPCS in (X, τ). This implies that (X, τ) is not an BIVMFGSP connected space. This is a contradiction. Therefore (X, τ) must be an BIVMFC$_5$- connected space.

Example 2.3. Let X = { a, b } and G = { ⟨ a, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.6, −0.6], [−0.7, −0.7], [−0.8, −0.8] ⟩, ⟨ b, [0.6, 0.6], [0.7, 0.7], [0.8,0.8], [−0.7, −0.7], [−0.8, −0.8], [−0.9, −0.9] ⟩}. Then τ = {0_X, G, 1_X } is an BIVMFT on X. Then X is an BIVMFC$_5$-connected space but not an BIVMFGSP connected space, since the BIVMFS A = { ⟨ a, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [−0.5, −0.5], [−0.6, −0.6], [−0.7, −0.7] ⟩, ⟨ b, [0.3, 0.3], [0.4, 0.4], [0.5, 0.5], [−0.4, −0.4], [−0.5, −0.5], [−0.6, −0.6] ⟩} in X is both an BIVMFGSPCS and an BIVMFGSPOS in X.

Theorem 2.4. Every BIVMFGSP connected space is an BIVMFGO-connected space.

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Proof. Let (X, τ) be an BIVMFGSP connected space. Suppose (X, τ) is not an BIVMFGO-connected space, then there exists a proper BIVMFS A which is both an BIVMFGOS and an BIVMFGCS in (X, τ). That is, A is both an BIVMFGSPOS and an BIVMFGSPCS in (X, τ). This implies that (X, τ) is not an BIVMFGSP connected space. This is a contradiction. Therefore (X, τ) must be an BIVMFGO-connected space.

Theorem 2.5. The BIVMFT (X, τ) is an BIVMFGSP connected space if and only if there exists no non-zero BIVMFGSPOS A and B in (X, τ) such that $A = B^c$.

Proof. Necessity. Assume that (X, τ) is an BIVMFGSP connected space. Let A and B be two BIVMFGSPOS in (X, τ) such that $A \neq 0_X \neq B$ and $A = B^c$. Therefore B^c is an BIVMFGSPCS. Since $A \neq 0_X$, $B \neq 1_X$. This implies B is a proper BIVMFS which is both an BIVMFGSPOS and an BIVMFGSPCS in (X, τ). Hence (X, τ) is not an BIVMFGSP connected space. But this is a contradiction to our hypothesis. Thus there exists no non-zero BIVMFGSPOSs A and B in (X, τ) such that $A = B^c$.

Sufficiency. Let A be both an BIVMFGSPOS and an BIVMFGSPCS in (X, τ) such that $1_X \neq A \neq 0_X$. Now let $B = A^c$. Then B is an BIVMFGSPOS and $B \neq 1_X$. This implies $B = A^c \neq 0_X$ which is a contradiction to our hypothesis. Therefore (X, τ) is an BIVMFGSP connected space.

Theorem 2.6. An BIVMFT (X, τ) is an BIVMFGSP connected space if and only if there exists no non-zero BIVMFGSPOSs A and B in (X, τ) such that $B = A^c$, $B = (\text{bivmfspcl}(A))^c$, $A = (\text{bivmfspcl}(B))^c$.

Proof. Necessity. Assume that there exist BIVMFS A and B such that $A \neq 0_X \neq B$, $B = A^c$, $B = (\text{bivmfspcl}(A))^c$, $A = (\text{bivmfspcl}(B))^c$. Since $(\text{bivmfspcl}(A))^c$ and $(\text{bivmfspcl}(B))^c$ are BIVMFGSPOSs in (X, τ), A and B are BIVMFGSPOSs in (X, τ). This implies (X, τ) is not an BIVMFGSP connected space, which is a contradiction. Therefore there exists no non-zero BIVMFGSPOSs A and B in (X, τ) such that $B = A^c$, $B = (\text{bivmfspcl}(A))^c$, $A = (\text{bivmfspcl}(B))^c$.

Sufficiency. Let A be both an BIVMFGSPOS and an BIVMFGSPCS in (X, τ) such that $1_X \neq A \neq 0_X$. Now by taking $B = A^c$, we obtain a contradiction to our hypothesis. Hence (X, τ) is an BIVMFGSP connected space.

Theorem 2.7. Let (X, τ) be an $\text{BIVMFSPT}^*_{1/2}$ space, then following statements are equivalent.
(i) (X, τ) is an BIVMFGSP connected space,
(ii) (X, τ) is an BIVMFGO-connected space,
(iii) (X, τ) is an BIVMFC_5-connected space.

Proof. (i) ⇒ (ii) is obvious from the Theorem 2.4.
(ii) ⇒(iii) is true.
(iii) ⇒ (i) Let (X, τ) be an BIVMFC_5–connected space. Suppose (X, τ) is not an BIVMFGSP connected space, then there exists a proper BIVMFS A in (X, τ) which is both an BIVMFGSPOS and an BIVMFGSPCS in (X, τ). But since (X, τ) is an $\text{BIVMFSPT}^*_{1/2}$ space, A is both an BIVMFOS and an BIVMFCS in (X, τ). This implies that (X, τ) is not an

BIVMFC$_5$-connected space, which is a contradiction to our hypothesis. Therefore (X, τ) must be an BIVMFGSP connected space.

Theorem 2.8. If f : (X, τ) → (Y, σ) is an BIVMFGSP continuous surjection and (X, τ) is an BIVMFGSP connected space, then (Y, σ) is an BIVMFC$_5$–connected space.

Proof. Let (X, τ) be an BIVMFGSP connected space. Suppose (Y, σ) is not an BIVMFC$_5$–connected space, then there exists a proper BIVMFS A which is both an BIVMFOS and an BIVMFCS in (Y, σ). Since f is an BIVMFGSP continuous mapping, $f^{-1}(A)$ is both an BIVMFGSPOS and an BIVMFGSPCS in (X, τ). But this is a contradiction to our hypothesis. Hence (Y, σ) must be an BIVMFC$_5$–connected space.

Theorem 2.9. If f : (X, τ) → (Y, σ) is an BIVMFGSP irresolute surjection and (X, τ) is an BIVMFGSP connected space, then (Y, σ) is also an BIVMFGSP connected space.

Proof. Suppose (Y, σ) is not an BIVMFGSP connected space, then there exists a proper BIVMFS A such that A is both an BIVMFGSPOS and an BIVMFGSPCS in (Y, σ). Since f is an BIVMFGSP irresolute mapping, $f^{-1}(A)$ is both an BIVMFGSPOS and an BIVMFGSPCS in (X, τ). But this is a contradiction to our hypothesis. Hence (Y, σ) must be an BIVMFGSP connected space.

Definition 2.10. An BIVMFS (X, τ) is BIVMFGSP connected between two BIVMFS A and B if there is no BIVMFGSPOS E in (X, τ) such that A ⊆ E and E $\not{q}$ B.

Example 2.11. Let X = { a, b } and G = { ⟨ a, [0.3, 0.3], [0.4, 0.4], [0.5, 0.5], [–0.4, –0.4], [–0.5, –0.5], [–0.6, –0.6] ⟩, ⟨b, [0.2, 0.2], [0.3, 0.3], [0.4, 0.4], [–0.3, –0.3], [–0.4, –0.4], [–0.5, –0.5] ⟩}. Then τ = { 0_X, G, 1_X } is an BIVMFT on X. Let A = { ⟨ a, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [–0.6, –0.6], [–0.7, –0.7], [–0.8, –0.8] ⟩, ⟨b, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [–0.5, –0.5], [–0.6, –0.6], [–0.7, –0.7] ⟩ } and B = { ⟨ a, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [–0.6, –0.6], [–0.7, –0.7], [–0.8, –0.8] ⟩, ⟨b, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [–0.5, –0.5], [–0.6, –0.6], [–0.7, –0.7] ⟩ } be two BIVMFS in X. Hence (X, τ) is BIVMFGSP connected between the two BIVMFS A and B.

Theorem 2.12. If an BIVMFT (X, τ) is BIVMFGSP connected between two BIVMFS A and B, then it is BIVMFC$_5$-connected between A and B but the converse may not be true in general.

Proof. Suppose (X, τ) is not BIVMFC$_5$-connected between A and B, then there exists an BIVMFOS E in (X, τ) such that A ⊆ E and E $\not{q}$ B. Since every BIVMFOS is an BIVMFGSPOS, there exists an BIVMFGSPOS E in (X, τ) such that A ⊆ E and E $\not{q}$ B. This implies (X, τ) is not BIVMFGSP connected between A and B, a contradiction to our hypothesis. Therefore (X, τ) must be BIVMFC$_5$-connected between A and B.

Example 2.13. Let X = { a, b } and G = { ⟨ a, [0.3, 0.3], [0.4, 0.4], [0.5, 0.5], [–0.4, –0.4], [–0.5, –0.5], [–0.6, –0.6] ⟩, ⟨b, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [–0.6, –0.6], [–0.7, –0.7], [–0.8, –0.8] ⟩, ⟨ b, [0.4, 0.4], [0.6, 0.6] ⟩ }. Then τ = { 0_X, G, 1_X } is an BIVMFT on X. Let A = { ⟨ a, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [–0.6, –0.6], [–0.7, –0.7], [–0.8, –0.8] ⟩, ⟨b, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [–0.5, –0.5], [–0.6, –0.6], [–0.7, –0.7] ⟩ } and B = { ⟨ a, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [–0.6, –0.6], [–0.7, –0.7], [–0.8, –0.8] ⟩, ⟨b, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6],

[−0.5, −0.5], [−0.6, −0.6], [−0.7, −0.7] ⟩ } be two BIVMFS in X. Then X is BIVMFC$_5$-connected between A and B, since there exists no BIVMFOS E in X such that A $\subseteq$ E and E q B. But it is not BIVMFGSP connected between the two BIVMFS A and B, since there exists an BIVMFGSPOS E = { ⟨ a, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.6, −0.6], [−0.7, −0.7], [−0.8, −0.8] ⟩, ⟨b, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [−0.5, −0.5], [−0.6, −0.6], [−0.7, −0.7] ⟩ } in X such that A $\subseteq$ E and E $\not q$ B.

Theorem 2.14. An BIVMFT (X, τ) is BIVMFGSP connected between two BIVMFS A and B if and only if there is no BIVMFGSPOS and BIVMFGSPCS E in (X, τ) such that A $\subseteq$ E $\subseteq$ B^c.

Proof. **Necessity.** Let (X, τ) be BIVMFGSP connected between A and B. Suppose that there exists an BIVMFGSPOS and an BIVMFGSPCS E in (X, τ) such that A $\subseteq$ E $\subseteq$ B^C, then E $\not q$ B and A $\subseteq$ E. This implies (X, τ) is not BIVMFGSP connected between A and B, by Definition 2.10. A contradiction to our hypothesis. Therefore there exists no BIVMFGSPOS and an BIVMFGSPCS E in (X, τ) such that A $\subseteq$ E $\subseteq$ B^C.

Sufficiency. Suppose that (X, τ) is not BIVMFGSP connected between A and B. Then there exists an BIVMFGSPOS E in (X, τ) such that A $\subseteq$ E and E $\not q$ B. This implies that there exists an BIVMFGSPOS E in (X, τ) such that A $\subseteq$ E $\subseteq$ B^C. But this is a contradiction to our hypothesis. Hence (X, τ) must be BIVMFGSP connected.

Theorem 2.15. If an BIVMFT (X, τ) is BIVMFGSP connected between A and B and A $\subseteq$ A_1, B $\subseteq$ B_1, then (X, τ) is BIVMFGSP connected between A_1 and B_1.

Proof. Suppose that (X, τ) is not BIVMFGSP connected between A_1 and B_1, then by Definition 2.10, there exists an BIVMFGSPOS E in (X, τ) such that A_1 $\subseteq$ E and E $\not q$ B_1. This implies E $\subseteq$ $B_1{}^c$ and between A and B. A_1 $\subseteq$ E implies A $\subseteq$ A_1 $\subseteq$ E. That is A $\subseteq$ E. Now let us prove that E $\subseteq$ B^C, that is let us prove E $\not q$ B. Suppose that E q B, then by Definition 1.1.20, there exists an element x in X such that $\mu_E(x) > \nu_B(x)$ and $\nu_E(x) < \mu_B(x)$. Therefore $\mu_E(x) > \nu_B(x) > \nu_{B1}(x)$ and $\nu_E(x) < \mu_B(x) < \mu_{B1}(x)$, since B $\subseteq$ B_1. Hence $\mu_E(x) > \nu_{B1}(x)$ and $\nu_E(x) < \mu_{B1}(x)$. Thus E q B_1. But E $\subseteq$ $B_1{}^c$. That is E $\not q$ B_1, which is a contradiction. Therefore E $\not q$ B. That is E $\subseteq$ B^c. Hence (X, τ) is not BIVMFGSP connected between A and B, which is a contradiction to our hypothesis. Thus (X, τ) must be BIVMFGSP connected between A_1 and B_1.

Theorem 2.16. Let (X, τ) be an BIVMFT and A and B be BIVMFS in (X, τ). If A q B, then (X, τ) is BIVMFGSP connected between A and B.

Proof. Suppose (X, τ) is not BIVMFGSP connected between A and B. Then there exists an BIVMFGSPOS E in (X, τ) such that A $\subseteq$ E and E $\subseteq$ B^c. This implies that A $\subseteq$ B^c. That is A $\not q$ B. But this is a contradiction to our hypothesis. Therefore (X, τ) is must be BIVMFGSP connected between A and B.

Definition 2.17. An BIVMFGSPOS A is an bipolar interval valued multi fuzzy regular generalized semipreopen set (BIVMFRGSPOS for short) if A = bivmfgspint (bivmfgspcl(A)). The complement of an BIVMFRGSPOS is called an bipolar interval valued multi fuzzy regular generalized semipreclosed set (BIVMFRGSPCS for short).

Definition 2.18. An BIVMFT (X, τ) is called an bipolar interval valued multi fuzzy generalized semipre (BIVMFGSP for short) super connected if there exists no proper BIVMFRGSPOS in (X, τ).

Theorem 2.19. Let (X, τ) be an BIVMFT. Then the following statements are equivalent.
(i) (X, τ) is an BIVMFGSP super connected space,
(ii) for every non-zero BIVMFRGSPOS A, bivmfgspcl(A) = 1_X,
(iii) for every BIVMFRGSPCS A with $A \neq 1_X$, bivmfgspint(A) = 0_X,
(iv) there exists no BIVMFRGSPOS A and B in (X, τ) such that $A \neq 0_X \neq B$, $A \subseteq B^c$,
(v) there exists no BIVMFRGSPOS A and B in (X, τ) such that $A \neq 0_X \neq B$, B = (bivmfgspcl(A))^c, A = (bivmfgspcl(B))^c,
(vi) there exists no BIVMFRGSPCS A and B in (X, τ) such that $A \neq 1_X \neq B$, B = (bivmfgspint(A))^c, A = (bivmfgspint(B))^c.

Proof. (i) ⇒ (ii) Assume that there exists an BIVMFRGSPOS A in (X, τ) such that $A \neq 0_X$ and bivmfgspcl(A) ≠ 1_X. Now let B = bivmfgspint(bivmfgspcl(A))^c. Then B is a proper BIVMFRGSPOS in (X, τ). But this is a contradiction to the fact that (X, τ) is an BIVMFGSP super connected space. Therefore bivmfgspcl(A) = 1_X.
(ii) ⇒ (iii) Let $A \neq 1_X$ be an BIVMFRGSPCS in (X, τ). If $B = A^c$, then B is an BIVMFRGSPOS in (X, τ) with $B \neq 0_X$. Hence bivmfgspcl(B) = 1_X. This implies (bivmfgspcl(B))^c = 0_X. That is bivmfgspint(B^c) = 0_X. Hence bivmfgspint(A) = 0_X.
(iii) ⇒ (iv) Suppose A and B be two BIVMFRGSPOSs in (X, τ) such that $A \neq 0_X \neq B$ and $A \subseteq B^c$. Then B^c is an BIVMFRGSPCS in (X, τ) and $B \neq 0_X$ implies $B^c \neq 1_X$. Therefore B^c = bivmfgspcl(bivmfgspint(B^c)). Also by hypothesis bivmfgspint(B^c) = 0_X. But $A \subseteq B^c$. Therefore $0_X \neq A$ = bivmfgspint(bivmfgspcl(A)) ⊆ bivmfgspint(bivmfgspcl(B^c)) = bivmfgspint(bivmfgspcl (bivmfgspcl (bivmfgspint(B^c)))) = bivmfgspint(bivmfgspcl (bivmfgspint(B^c))) = bivmfgspint(B^c) = 0_X. A contradiction arises. Therefore (iv) is true.
(iv) ⇒ (i) Suppose $0_X \neq A \neq 1_X$ be an BIVMFRGSPOS in (X, τ). If we take B = (bivmfgspcl(A))^c, then we have, B is an BIVMFRGSPOS, since bivmfgspint (bivmfgspcl(B)) = bivmfgspint (bivmfgspcl (bivmfgspcl (A))^c) = bivmfgspint (bivmfgspint (bivmfgspcl (A)))^c = bivmfgspint (A^c) = (bivmfgspcl(A))^c = B.Also we get $B \neq 0_X$, since otherwise, if we have $B = 0_X$, then this implies (bivmfgspcl(A))^c = 0_X. That is bivmfgspcl(A) = 1_X. Hence A = bivmfgspint (bivmfgspcl(A)) = bivmfgspint(1_X) = 1_X. That is A = 1_X which is a contradiction. Therefore $B \neq 0_X$ and $A \subseteq B^c$. But this is a contradiction to (iv). Therefore (X, τ) must be an BIVMFGSP super connected space.
(i)⇒ (v) Suppese A and B be two BIVMFRGSPOSs in (X, τ) such that $A \neq 0_X \neq B$ and B = (bivmfgspcl(A))^c, A = (bivmfgspcl(B))^c. Now we have bivmfgspint(bivmfgspcl(A)) = bivmfgspint(B^c) = bivmfgspcl(B))^c = A, $A \neq 0_X$ and $A \neq 1_X$, since if $A = 0_X$, then 1_X = (bivmfgspcl(B))^c ⇒ bivmfgspcl(B) = 0_X ⇒ B = 0_X. But $B \neq 0_X$. Therefore $A \neq 1_X$. That is, A is a proper BIVMFRGSPOS in (X, τ), which is a contradiction to (i). Hence (v) is true.
(v) ⇒ (i) Let A be an BIVMFRGSPOS in (X, τ) such that A = bivmfgspint (bivmfgspcl(A)), $0_X \neq A \neq 1_X$. Now take B = (bivmfgspcl(A))^c. In this case we get,$B \neq 0_X$ and B is an BIVMFRGSPOS in (X, τ). Now B = (bivmfgspcl(A))^c and (bivmfgspcl(B))^c = (bivmfgspcl (bivmfgspcl(A))^c)^c = bivmfgspint(bivmfgspcl(A)^c)^c = bivmfgspint (bivmfgspcl(A)) = A. But this is a contradiction to (v). Therefore (X, τ) must be an BIVMFGSP super connected space.
(v) ⇒ (vi) Suppose A and B be two BIVMFRGSPCS in (X, τ) such that $A \neq 1_X \neq B$, B = (bivmfgspint(A))^c, A = (bivmfgspint(B))^c. Taking $C = A^c$ and $D = B^c$, C and D become

BIVMFRGSPOSs in (X, τ) with C ≠ 0_X ≠ D and D = $(bivmfgspcl(C))^c$, C = $(bivmfgspcl(D))^c$, which is a contradiction to (v). Hence (vi) is true.
(vi) ⇒ (v) can be proved easily by the similar way as in (v)⇒ (vi).

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