Coloring Fuzzy Graphs Using Strong Alpha Matching

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Abstract — **In this paper we define strong alpha matching for finding the chromatic number of fuzzy graphs. The number of elements in the strong alpha matching is equal to the chromatic number. Also we found an application of strong alpha matching to job assigning problem.**

Keywords— **Chromatic number, fuzzy set, fuzzy graph, Fuzzy matching, Strong alpha matching.**

I. Introduction

Zadeh introduced the notion of fuzzy sets and fuzzy relations to deal with the problems of uncertainty in real life situations [9]. In 1973 Kaufmann was first defined the fuzzy graph [2] based on Zadeh's Fuzzy relation [8]. Later in 1975 Rosenfeld defined the application of fuzzy graph in cognitive decision process. In [3] O.T. Manjusha and M.S. Sunitha found coverings, matchings and paired domination in fuzzy graphs using strong arcs. Also they have obtained the relationship between covering and matchings analogues to Gallai's results. In [7] R.Seethalakshmi and R.B.Gnanajothi derived a necessary condition for a fuzzy graph on a cycle or a complete graph or a star graph to have a perfect fuzzy matching. Also they have discussed perfect fuzzy matching on strong regular fuzzy graphs. In this paper we define the strong alpha matching for finding the chromatic number of fuzzy graphs. Also, we have solved job assigning problem [1] using these strong alpha matching.

Page Layout

Your paper must use a page size corresponding to A4 which is 210mm (8.27") wide and 297mm (11.69") long. The margins must be set as follows:

- Top = Bottom= 19mm (0.75")
- Left = Right = 14.32mm (0.56")

II. Page Style

I. Preliminaries

2.1 Definition

A Fuzzy set A defined on a non empty set X is the family A= $\{(x, \mu_A(x) / x \in X\}$ where μ_A: X→I is the membership function defined on X to the interval [0, 1]

In other words, A fuzzy set of a base set (or reference set) X is specified by its membership function σ, where σ: X→ [0, 1] assigning to each $x \in X$ the degree or grade to which x belongs to σ.

2.2 Definition

A fuzzy graph G=(σ, μ) is a pair of function σ:V→[0,1] and μ:V x V→[0,1],where for all $u, v \in V$, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

The following figure 1 is the example of fuzzy graph.

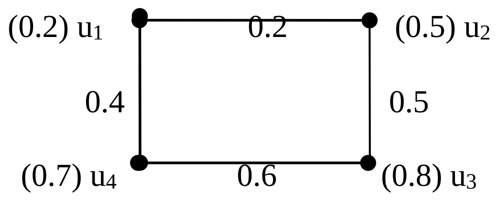

Figure. 1

Here σ (u_1) =0.2, σ (u_2) = 0.5, σ (u_3) = 0.7, σ (u_4) = 0.8
μ (u_1, u_2)= 0.2, μ (u_2, u_3)=0.4, μ (u_3, u_4) = 0.6, μ (u_4, u_1) = 0.5

2.3 Definition

The α-cut of a fuzzy set A is the crisp set ${}^{\alpha}A$ that contains all the elements of the universal set X whose membership grades in A are greater than or equal to the specified value of α. It is denoted by ${}^{\alpha}A = \{x / A(x) \geq \alpha\}$.

2.4 **Definition**

Let G= (σ, μ) be a fuzzy graph on (V, E) where V is the vertex set and E is the set of edges with membership grade in the interval [0, 1]. A subset M of E is called a Fuzzy matching if for each vertex u, we have $\sum_{u \in v} \mu(u,v) \leq \sigma(u)$.

Inthe following figure 2 is the example of fuzzy matching. In this fuzzy graph V={v1,v2,v3,v4} and E={e1,e2,e3,e4} with e1=v1v2, e2=v3v4, e3=v3v4, e4=v4v1

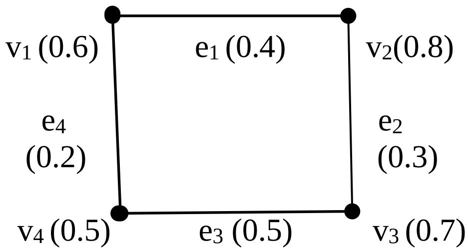

Figure. 2

$If\ u = v_1$

$$\sum_{v_1 \in V} \mu(v_1, v_2) = 0.4 + 0.2 = 0.6 \leq \sigma(u)$$

$If\ u = v_2$

$$\sum_{v_2 \in V} \mu(v_1, v_4) = 0.4 + 0.3 = 0.7 \leq \sigma(u)$$

$If\ u = v_3$

$$\sum_{v_3 \in V} \mu(v_3, v_4) = 0.3 + 0.5 = 0.8 \neq \sigma(u)$$

$If\ u = v_4$

$$\sum_{v_1 \in V} \mu(v_4, v_1) = 0.5 + 0.2 = 0.7 \neq \sigma(u)$$

From the above result we obtain σ (v1) and σ (v2) are the fuzzy matchings of the given fuzzy graph.

2.5 Definition

If G = (V, σ, □) is such a fuzzy graph where V={1, 2, 3,..., n} and □ is a fuzzy number on the set of all subsets of V x V ,$G_\alpha$denote the crisp graph $G_\alpha = (V, E_\alpha)$ where

$E_\alpha = \{ij/1 \le i < j \le n,\ \mu(i,j) \ge \alpha\}$ and χ_α=□(G_α) denote the chromatic number of crisp graph G_α. By this definition the chromatic number of fuzzy graph G is the fuzzy number
□(G) $= \{(i, v(i))/i \in X\}$ where $V(i) = max\ \{\alpha \in I/i \in A_\alpha\}$ and $A_\alpha = (1,2,3,\ldots,\chi_\alpha)$.

II. Strong alpha matching

3.1 Definition

A fuzzy matching M is called strong alpha matching (SAM) if for each vertex u, $\sum_{u \in v} {}_{u,v \in M}\ \mu_\alpha(u,v) = \sigma(u) > \alpha^+$.

In this paper the element of SAM is equal to the chromatic number of fuzzy graph G is denoted by $\chi_{SAM}(G)$.

3.2 Example

Consider the fuzzy graph in figure 3,

Here V= {v_1, v_2, v_3, v_4} and E= {e_1, e_2, e_3, e_4} with $e_1=v_1v_2$, $e_2= v_2v_3$, $e_3=v_3v_4$, $e_4= v_4v_1$

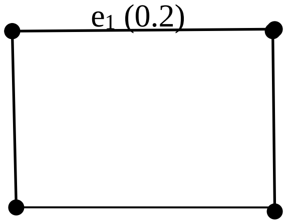

Figure. 3

If α=0.2

$$\sum_{v_1 \in V} {}_{(v_1,v_4) \in M}\ \mu_\alpha(v_1, v_4) = 0.2 + 0.3 = 0.5 = \sigma(v_1) > \alpha^+.$$

If α=0.3

$$\sum_{v_3 \in V} {}_{(v_3,v_4) \in M}\ \mu_\alpha(v_3, v_4) = 0.6 + 0.2 = 0.8 = \sigma(v_3) > \alpha^+.$$

$$\sum_{v_3 \in V} {}_{(v_3,v_4) \in M}\ \mu_\alpha(v_3, v_4) = 0.6 + 0.4 = 1 \ne \sigma(v_4) > \alpha^+.$$

If α=0.4

$$\sum_{v_3 \in V} {}_{(v_2,v_3) \in M}\ \mu_\alpha(v_2, v_3) = 0.2 + 0.6 = 0.8 = \sigma(v_3) > \alpha^+.$$

If α=0.5

$$\sum_{v_3 \in V} {}_{(v_2,v_3) \in M}\ \mu_\alpha(v_2, v_3) = 0.4 + 0.6 = 1.0 \ne \sigma(v_3).$$

$Here\ \sigma(v_1) and\ \sigma(v_3) > \alpha^+.$

$From\ above vertices\ we\ have (v_1, v_2) = e_1, (v_3, v_4) = e_2,$

Therefore the strong alpha matchings are e1 and e2

3.3 Theorem

In a fuzzy graph G= (σ, μ) have a strong alpha matching if and only if every strong alpha matching is maximum.

Proof:

Let M be a matching in fuzzy graph G and M is a subset of E.

By definition of strong alpha matching each and every matching value is greater than α. It is denoted by α+. Hence every strong alpha matching is maximum.

Conversely, by the definition of strong alpha in the fuzzy graph, alpha value is maximum than σ (u) then there exists a strong alpha matching between the vertices. Hence the proof.

3.4 Example

Consider the fuzzy graph in figure 4, Here $V=\{v_1,v_2,v_3,v_4\}$ and $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$ with $e_1=v_1v_2$, $e_2= v_2v_3$, $e_3=v_3v_4$, $e_4= v_4v_1$, $e_6= v_3v_1$, $e_5 = v_4v_2$

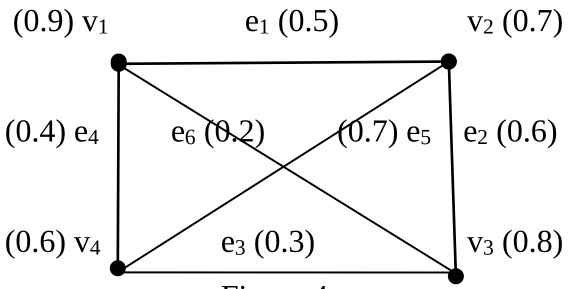

Figure.4

If α=0.2

$$\sum_{v_1\in V}\ _{(v_1,v_2)\in M}\ \mu_\alpha(v_1,v_2)=0.4+0.5=0.9=\sigma(v_1)>\alpha^+.$$

$$\sum_{v_3\in V}\ _{(v_3,v_4)\in M}\ \mu_\alpha(v_3,v_4)=0.6+0.2=0.8=\sigma(v_3)>\alpha^+.$$

$$\sum_{v_4\in V}\ _{(v_4,v_1)\in M}\ \mu\alpha(v_4,v_1)=0.2+0.4=0.6=\sigma(v_4)>\alpha^+$$

If α=0.3

$$\sum_{v_4\in V}\ _{(v_3,v_4)\in M}\ \mu_\alpha(v_4,v_3)=0.4+0.3=0.7\neq\sigma(v_4)>\alpha^+.$$

$$\sum_{v_3\in V}\ _{(v_3,v_4)\in M}\ \mu_\alpha(v_3,v_4)=0.6+0.3=0.9\neq\sigma(v_3)>\alpha^+.$$

If α=0.4

$$\sum_{v_1\in V}\ _{(v_1,v_2)\in M}\ \mu_\alpha(v_1,v_2)=0.4+0.5=0.9=\sigma(v_1)>\alpha^+.$$

$$\sum_{v_4\in V}\ _{(v_4,v_2)\in M}\ \mu_\alpha(v_4,v_2)=0.4+0.7=1.1\neq\sigma(v_4)$$

If α=0.5

$$\sum_{v_1\in V}\ _{(v_1,v_2)\in M}\ \mu_\alpha(v_1,v_2)=0.4+0.5=0.9=\sigma(v_1)>\alpha^+.$$

If α=0.6

$$\sum_{v_2 \in V} \, _{(v_2, v_3) \in M} \mu_\alpha(v_2, v_3) = 0.5 + 0.6 = 1.1 \neq \sigma(v_2).$$

$$\sum_{v_3 \in V} \, _{(v_3, v_1) \in M} \mu_\alpha(v_3, v_1) = 0.6 + 0.2 = 0.8 = \sigma(v_3) > \alpha^+.$$

If α=0.7

$$\sum_{v_4 \in V} \, _{(v_4, v_2) \in M} \mu_\alpha(v_4, v_2) = 0.7 + 0.3 = 1.0 \neq \sigma(v_4).$$

$$\sum_{v_2 \in V} \, _{(v_2, v_4) \in M} \mu_\alpha(v_2, v_4) = 0.7 + 0.6 = 1.3 \neq \sigma(v_3)$$

$Here\, \sigma(v_1), \sigma(v_3)\, and\, \sigma(v_4) > \alpha^+.$

$From\, above\, vertices\, we\, have\, (v_1, v_2) = e_1, (v_1, v_3) = e_6,$

$(v_1, v_4) = e_4\, and\, (v_2, v_3) = e_2$

Therefore the strong alpha matchings are e1, e2, e4 and e6. This example proves the above theorem.

iv. Job assinging problem

In a company there are six workers and five jobs. So they decided to minimize the workers and maximize their profit [1]. In this situation they decide some of the workers were well versed in two different jobs, that workers were able to do the jobs at simultaneously. Hence the company decides to retain that kind of workers alone for the production of material. In this situation we can apply the concept of coloring using strong alpha matching.

We design this situation as a fuzzy graph, the jobs are considered as vertices and the workers are considered as edges. Since each edge have two vertices. We give a membership values for both vertices and edges to apply a fuzzy matching concept for find a strong alpha matching. Since the number of elements of strong alpha matching is considered as the chromatic number of the fuzzy graph. In the following figure 5 illustrate the situation of the company.

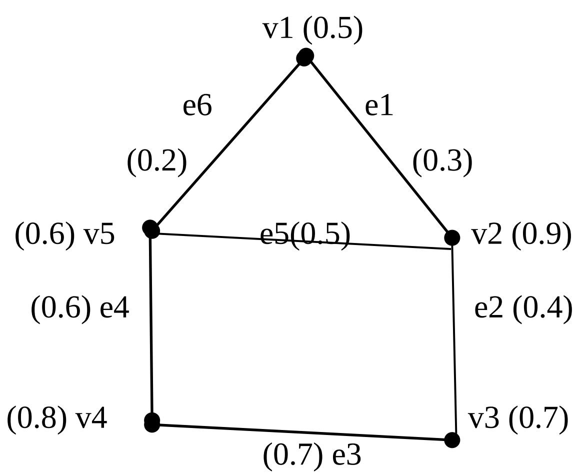

Figure.5

If α=0.2

$\sum_{v_2 \in V} (v_2,v_1) \in M\ \mu_\alpha(v_2,v_1) = 0.2+0.4 = 0.6 \neq \sigma(v_2).$

$\sum_{v_2 \in V} (v_2,v_5) \in M\ \mu_\alpha(v_2,v_5) = 0.5+0.4 = 0.9 = \sigma(v_2) > \alpha^+.$

$\sum_{v_2 \in V} (v_2,v_3) \in M\ \mu_\alpha(v_2,v_3) = 0.4+0.7 = 1.1 \neq \sigma(v_2)$

$\sum_{v_1 \in V} (v_1,v_5) \in M\ \mu_\alpha(v_1,v_5) = 0.2+0.3 = 0.5 = \sigma(v_1) > \alpha^+$

$\sum_{v_5 \in V} (v_5,v_1) \in M\ \mu_\alpha(v_5,v_1) = 0.6+0.3 = 0.9 \neq \sigma(v_5)$

$\sum_{v_2 \in V} (v_2,v_3) \in M\ \mu_\alpha(v_2,v_3) = 0.5+0.4 = 0.9 = \sigma(v_2) > \alpha^+.$

$\sum_{v_3 \in V} (v_3,v_2) \in M\ \mu_\alpha(v_3,v_2) = 0.4+0.7 = 1.1 \neq \sigma(v_3)$

$\sum_{v_4 \in V} (v_4,v_5) \in M\ \mu_\alpha(v_4,v_5) = 0.6+0.7 = 1.3 \neq \sigma(v_4)$

$\sum_{v_5 \in V} (v_5,v_4) \in M\ \mu_\alpha(v_5,v_4) = 0.6+0.5 = 1.1 \neq \sigma(v_5)$

If α=0.3

$\sum_{v_5 \in V} (v_5,v_1) \in M\ \mu_\alpha(v_5,v_1) = 0.3+0.6 = 0.9 \neq \sigma(v_5).$

$\sum_{v_2 \in V} (v_2,v_3) \in M\ \mu\alpha(v_2,v_3) = 0.4+0.5 = 0.9 = \sigma(v_2) > \alpha^+$

$\sum_{v_5 \in V} (v_5,v_2) \in M\ \mu_\alpha(v_5,v_2) = 0.5+0.6 = 1.1 \neq \sigma(v_5)$

$\sum_{v_3 \in V} (v_3,v_2) \in M\ \mu_\alpha(v_3,v_2) = 0.4+0.7 = 1.1 \neq \sigma(v_3)$

$\sum_{v_4 \in V} (v_4,v_5) \in M\ \mu_\alpha(v_4,v_5) = 0.7+0.6 = 1.3 \neq \sigma(v_4)$

If α=0.4

$\sum_{v_2 \in V} (v_2,v_5) \in M\ \mu_\alpha(v_2,v_5) = 0.42+0.5 = 0.9 = \sigma(v_2) > \alpha^+$

$\sum_{v_3 \in V} (v_3,v_4) \in M\ \mu_\alpha(v_3,v_4) = 0.4+0.7 = 1.1 \neq \sigma(v_3).$

$\sum_{v_4 \in V} (v_4,v_3) \in M\ \mu_\alpha(v_4,v_3) = 0.7+0.6 = 1.3 \neq \sigma(v_4).$

$\sum_{v_5 \in V} (v_5,v_4) \in M\ \mu_\alpha(v_5,v_4) = 0.5+0.6 = 1.1 \neq \sigma(v_5).$

If α=0.5

$\sum_{v_4 \in V} (v_4,v_5) \in M\ \mu_\alpha(v_4,v_5) = 0.7+0.6 = 1.3 \neq \sigma(v_4).$

$\sum_{v_5 \in V} (v_5,v_2) \in M\ \mu_\alpha(v_5,v_2) = 0.6+0.5 = 1.1 \neq \sigma(v_5).$

If α=0.6

$\sum_{v_4 \in V} (v_4,v_5) \in M\ \mu_\alpha((v_4,v_5) = 0.6+0.7 = 1.3 \neq \sigma(v_4).$

If α=0.7

$\sum_{v_4 \in V} (v_5,v_4) \in M\ \mu_\alpha(v_5,v_4) = 0.7+0.6 = 1.3 \neq \sigma(v_4).$

Here σ (v1) and, σ (v2) >α+. The edge set of strong alpha matchings are e1, e2 and e6 which are equal to the alpha values of given fuzzy graph. Therefore the chromatic number is 3. Hence the company will decide to retain three workers only, because they are able to do two works at simultaneously.

III. CONCLUSION

In this paper we define a strong alpha matching, theorem with examples and an application of strong alpha matching in job assigning problem for finding the chromatic number to get the maximum profit in minimum number of workers. In further we can use this concept in many real life situations like face authentication, genetic algorithms and neural networks.

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